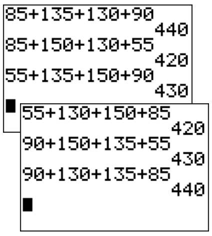
1a Er zijn $3 \cdot 2 \cdot 1 \cdot 1 = 6$ rondritten mogelijk. (het maakt niet uit waar de rondrit begint)
Er zijn steeds twee rondritten met gelijke lengte (maar in omgekeerde volgorde).

1b



```
85+135+130+90
              440
85+150+130+55
              420
55+135+150+90
              430
```

```
55+130+150+85
              420
90+150+135+55
              430
90+130+135+85
              440
```

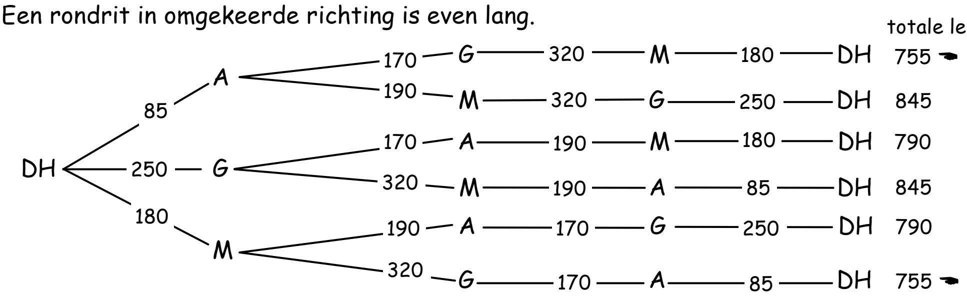
Utrecht-Eindhoven-Zwolle-Haarlem-Utrecht en Utrecht-Haarlem-Zwolle-Eindhoven-Utrecht zijn de kortste.

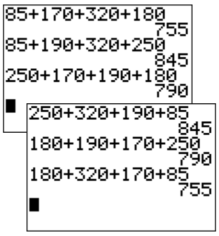
2 De grafen b en d zijn gelijkwaardig.
(beide hebben 5 knooppunten; in 1 knooppunt komen 4 wegen en in elk van de andere 4 knooppunten komen 3 wegen)

3a Er zijn $3 \cdot 2 \cdot 1 \cdot 1 = 6$ rondritten mogelijk.

3b Een rondrit in omgekeerde richting is even lang.

3c





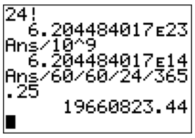
```
85+170+320+180
             755
85+190+320+250
             845
250+170+190+180
             790
```

```
250+320+190+85
             845
180+190+170+250
             790
180+320+170+85
             755
```

Advies: DH-A-G-M-DH of omgekeerd DH-M-G-A-DH.

4a Er zijn $24 \cdot 23 \cdot 22 \cdot ... \cdot 3 \cdot 2 \cdot 1 \cdot 1 = 24! \approx 6{,}20 \cdot 10^{23}$ rondritten mogelijk.
(vanuit elk punt kun je op 24 manieren starten, daarna op 23 manieren verder gaan, ...)

4b Het duurt bijna 20 miljoen jaar. (zie de berekening hiernaast)

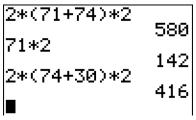


```
24!
  6.204484017ᴇ23
Ans/10^9
  6.204484017ᴇ14
Ans/60/60/24/365
.25
     19660823.44
```

5a De (kortste) afstand in km tussen N en R (via V) is $61 + 23 = 84$.
De (kortste) afstand in minuten tussen N en R (via 's H, E en W) is $29 + 21 + 17 + 14 = 81$.

5b De kosten van een treinkaartje enkele reis tweede klas.

6a Am → As: 2 (2× /week) $\cdot (71 + 74) \cdot 2$ (heen en terug) $= 580$ (km).
Zw → Am: 1 (1× /week) $\cdot 71 \cdot 2$ (heen en terug) $= 142$ (km).
Zw → Gr: 2 (2× /week) $\cdot (74 + 30) \cdot 2$ (heen en terug) $= 416$ (km).

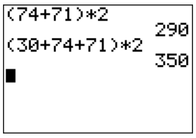


```
2*(71+74)*2
            580
71*2
            142
2*(74+30)*2
            416
```

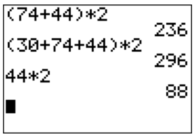
afstand per week van
centraal magazijn

$$\text{naar}\ \begin{array}{cccccc} & \text{Am} & \text{Ap} & \text{As} & \text{Gr} & \text{Zw} \\ \text{Am} & 0 & 86 & 290 & 350 & 142 \\ \text{Ap} & 86 & 0 & 236 & 296 & 88 \\ \text{As} & 580 & 472 & 0 & 120 & 296 \\ \text{Gr} & 700 & 592 & 120 & 0 & 416 \\ \hline\hline & 1366 & 1025 & 646 & 766 & 942 \end{array} = M$$

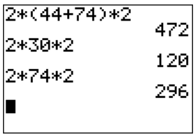
6b Zie de matrix M hiernaast. (heel wat gepuzzel en rekenwerk)



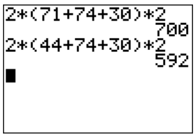
```
(74+71)*2
         290
(30+74+71)*2
         350
```



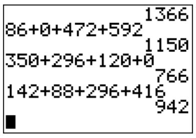
```
(74+44)*2
         236
(30+74+44)*2
         296
44*2
          88
```



```
2*(44+74)*2
         472
2*30*2
         120
2*74*2
         296
```

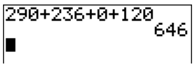


```
2*(71+74+30)*2
         700
2*(44+74+30)*2
         592
```



```
            1366
86+0+472+592
            1150
350+296+120+0
            766
142+88+296+416
            942
```

6c Tel alle afstanden van As naar alle andere vestigingen (kolom onder As) bij elkaar op. Dus $290 + 236 + 0 + 120 = 646$ (km).



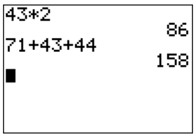
```
290+236+0+120
          646
```

6d Tel in elke kolom de getallen op (zie onder matrix M) ⇒ ADVIES: magazijn in Assen (646 km/week).

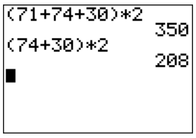
6e Am → Am → Ap: 1 (1× /week) $\cdot (0 + 43) \cdot 2$ (heen en terug) $= 86$ (km).
Zw → Am → Ap: 1 (1× /week) $\cdot (71 + 43 + 44)$ (andere weg terug) $= 158$ (km)
Am → As → Gr: 1 (1× /week) $\cdot (71 + 74 + 30) \cdot 2$ (heen en terug) $= 350$ (km).
Zw → As → Gr: 1 (1× /week) $\cdot (74 + 30) \cdot 2$ (heen en terug) $= 208$ (km)

centraal magazijn

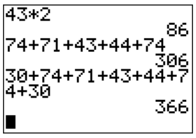
$$\text{naar}\ \begin{array}{cccccc} & \text{Am} & \text{Ap} & \text{As} & \text{Gr} & \text{Zw} \\ \text{Am-Ap} & 86 & 86 & 306 & 366 & 158 \\ \text{As-Gr} & 350 & 296 & 60 & 60 & 208 \\ \hline\hline & 436 & 382 & 366 & 426 & 366 \end{array} = N$$



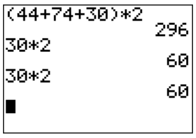
```
43*2
           86
71+43+44
          158
```



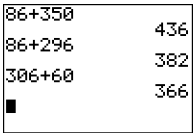
```
(71+74+30)*2
          350
(74+30)*2
          208
```



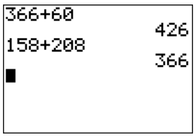
```
43*2
           86
74+71+43+44+74
          306
30+74+71+43+44+7
4+30
          366
```



```
(44+74+30)*2
          296
30*2
           60
30*2
           60
```



```
86+350
          436
86+296
          382
306+60
          366
```



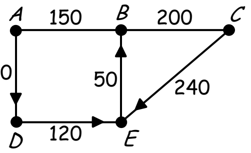
```
366+60
          426
158+208
          366
```

6f Tel in elke kolom de getallen op (zie onder matrix N) ⇒ ADVIES: magazijn in Assen of Zwolle (366 km/week).

7a Zie de graaf (punten en verbindingen) hiernaast.

7b "$A \to D$" = 40 en "$D \to A$" = 320.
Zie de verder ingevulde matrix M hiernaast.
(weer even puzzelen)

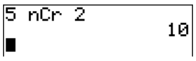
7c In deze matrix is: "$A \to D$" $\neq$ "$D \to A$".



$$\begin{array}{cc} & \text{van} \\ & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \text{naar} \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{pmatrix} 0 & 150 & 350 & 320 & 200 \\ 150 & 0 & 200 & 170 & 50 \\ 350 & 200 & 0 & 370 & 250 \\ 40 & 190 & 390 & 0 & 240 \\ 160 & 310 & 240 & 120 & 0 \end{pmatrix} = M \end{array}$$

8a De lijnstukken geven elk een verbinding aan tussen de plaatsen.

8b Telkens twee van de vijf punten kiezen, waarbij de volgorde niet van belang is.
<u>1 1 0 0 0</u> ("1" staat voor "wel" en "0" voor "niet") kan op $\binom{5}{2} = 5nCr2 = 10$ manieren $\Rightarrow$ 10 verbindingen.

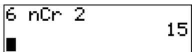


```
5 nCr 2
                10
■
```

8c Er zijn minimaal 4 verbindingen nodig om 5 steden onderling bereikbaar te laten zijn.
(denk de punten achter elkaar aan een snelweg)

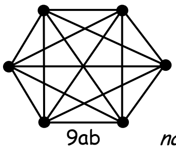
9a Verbind elk "knooppunt" met elk "ander knooppunt".
Zie de graaf en de verbindingsmatrix V hiernaast.

9b Een "maximale" graaf heeft $\binom{6}{2} = 6nCr2 = 15$ wegen.



```
6 nCr 2
                15
■
```

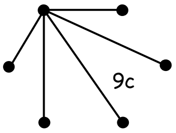
9c Een "minimale" graaf heeft $6 - 1 = 5$ wegen. (zie 2 voorbeelden hiernaast)

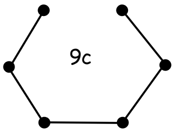


9ab

$$\begin{array}{cc} & \text{van} \\ & \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \text{naar} \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \end{array} & \begin{pmatrix} 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 1 & 0 \end{pmatrix} = V \end{array}$$

de hoofddiagonaal





10a Een graaf van n knooppunten die maximaal verbonden is, heeft $\binom{n}{2}$ wegen.

10b In de verbindingsmatrix van een graaf van n knooppunten die maximaal verbonden is, staan op de hoofddiagonaal nullen en op de andere plaatsen enen.

10c Een graaf van n knooppunten die minimaal verbonden is heeft $n - 1$ wegen.

11a Zie de matrix M hiernaast.

11b De grootste minimale afstand in M is 3 $\Rightarrow$ de diameter van de graaf in figuur 12.14 is 3.

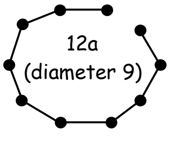
11c Zie de kolomtotalen onder matrix M. (een kleiner totaal betekent een betere bereikbaarheid)
De rangschikking (van bereikbaarheid) is E, A en F, B en D, C.

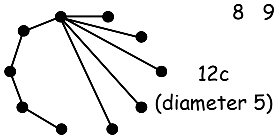
$$\begin{array}{cc} & \text{van} \\ & \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \text{naar} \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \\ \\ \end{array} & \begin{array}{c} \begin{pmatrix} 0 & 1 & 2 & 2 & 1 & 2 \\ 1 & 0 & 3 & 1 & 2 & 2 \\ 2 & 3 & 0 & 3 & 1 & 2 \\ 2 & 1 & 3 & 0 & 2 & 1 \\ 1 & 2 & 1 & 2 & 0 & 1 \\ 2 & 2 & 2 & 1 & 1 & 0 \end{pmatrix} = M \\ \hline \begin{array}{cccccc} 8 & 9 & 11 & 9 & 7 & 8 \end{array} \end{array} \end{array}$$

12a De maximale diameter is 9 (zie de graaf hiernaast).

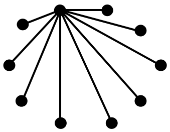
12b Bij maximale verbondenheid is elk punt met elk ander punt verbonden $\Rightarrow$ de diameter is dan 1.

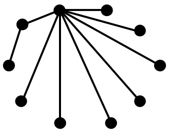
12c Zie de rechter graaf hiernaast.

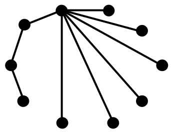


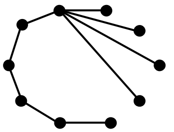


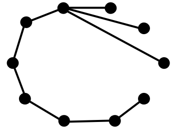
12d De diameter van een graaf met minimale verbondenheid kan 2, 3, 4, 5, 6, 7, 8 of 9 zijn (zie de grafen hier omheen).

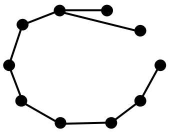

diameter 2


diameter 3

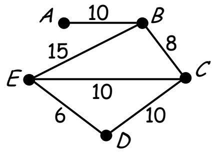

diameter 4


diameter 6


diameter 7

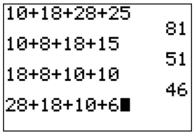

diameter 8

13a Teken eerst de graaf (zie hiernaast) met behulp van de verbindindingsmatrix V en de afstandenmatrix M.
Zet er de getallen van de matrix M bij.
$A \to E = 25$ (gegeven in M) $\Rightarrow B \to E = 10$.
($A \to E$ via C en D is $10 + 8 + 10 + 6 > 25$)
Nu is de matrix M verder in te vullen (zie hiernaast).



$$\begin{array}{ccc} & \text{van} & \\ & \begin{array}{ccccc} A & B & C & D & E \end{array} & \text{totaal} \\ \text{naar} \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{pmatrix} 0 & 10 & 18 & 28 & 25 \\ 10 & 0 & 8 & 18 & 15 \\ 18 & 8 & 0 & 10 & 10 \\ 28 & 18 & 10 & 0 & 6 \\ 25 & 15 & 10 & 6 & 0 \end{pmatrix} = M & \begin{array}{c} 81 \\ 51 \\ 46 \\ 62 \\ 56 \end{array} \end{array}$$

13b Tel nu de getallen per rij in matrix M op. (de totale afstanden van alle punten <u>naar</u> A, <u>naar</u> B enz.)
De totale afstand <u>naar</u> C is het kleinst $\Rightarrow$ het ziekenhuis komt in C.



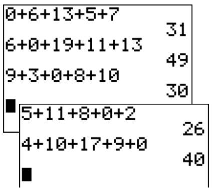
```
10+18+28+25
              81
10+8+18+15
              51
18+8+10+10
              46
28+18+10+6■
```

14a **verbindingsmatrix**

$$\begin{array}{cc} & \text{van} \\ & \begin{array}{ccccc} A & B & C & D & E \end{array} \\ \text{naar} \begin{array}{c} A \\ B \\ C \\ D \\ E \end{array} & \begin{pmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} = V \end{array}$$

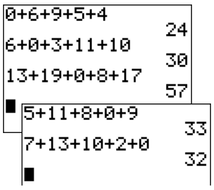
14b **afstandmatrix**

$$\begin{array}{ccc} & \text{van} & \\ & \begin{array}{ccccc} A & B & C & D & E \end{array} & \\ \text{naar} \begin{array}{c} A \\ B \\ C \\ D \\ E \\ \\ \end{array} & \begin{array}{c} \begin{pmatrix} 0 & 6 & 13 & 5 & 7 \\ 6 & 0 & 19 & 11 & 13 \\ 9 & 3 & 0 & 8 & 10 \\ 5 & 11 & 8 & 0 & 2 \\ 4 & 10 & 17 & 9 & 0 \end{pmatrix} = M \\ \hline \begin{array}{ccccc} 24 & 30 & 57 & 33 & 32 \end{array} \end{array} & \begin{array}{c} 31 \\ 49 \\ 30 \\ 26 \\ 40 \\ \\ \end{array} \end{array}$$



```
0+6+13+5+7
              31
6+0+19+11+13
              49
9+3+0+8+10
              30
■
```

```
5+11+8+0+2
              26
4+10+17+9+0
              40
■
```



```
0+6+9+5+4
              24
6+0+3+11+10
              30
13+19+0+8+17
              57
■
```

```
5+11+8+0+9
              33
7+13+10+2+0
              32
■
```

14c Zie 14b: $6+0+19+11+13$ (heenweg naar B) $+\,6+0+3+11+10$ (terugweg van B) $=49+30=79$ (km).

14d 31 (heenweg naar A) $+\,24$ (terugweg van A) $=55$ (km). 26 (heenweg naar D) $+\,33$ (terugweg van D) $=59$ (km).
30 (heenweg naar C) $+\,57$ (terugweg van C) $=87$ (km). 40 (heenweg naar E) $+\,32$ (terugweg van E) $=72$ (km). Dus in A.

15a **directe - wegenmatrix**

$$\text{naar}\quad \begin{array}{c} \text{van} \\ \begin{array}{c|ccccc|} & A & B & C & D & E \\ A & 1 & 1 & 0 & 0 & 0 \\ B & 1 & 0 & 2 & 0 & 0 \\ C & 0 & 2 & 0 & 0 & 3 \\ D & 0 & 0 & 2 & 0 & 1 \\ E & 0 & 0 & 3 & 1 & 0 \end{array} \end{array} = W$$

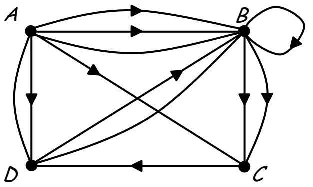
Alle 'niet nullen' in W worden enen in V.
(als er een weg is, is er een verbinding)

15b **verbindingsmatrix**

$$\text{naar}\quad \begin{array}{c} \text{van} \\ \begin{array}{c|ccccc|} & A & B & C & D & E \\ A & 1 & 1 & 0 & 0 & 0 \\ B & 1 & 0 & 1 & 0 & 0 \\ C & 0 & 1 & 0 & 0 & 1 \\ D & 0 & 0 & 1 & 0 & 1 \\ E & 0 & 0 & 1 & 1 & 0 \end{array} \end{array} = V$$

15c De diameter (het grootste aantal van de minimale stappen tussen elk tweetal punten) van de graaf is 4.
De diameter van de graaf in dit voorbeeld is dus het aantal stappen van D naar A.

16a



16b

$$\text{naar}\quad \begin{array}{c} \text{van} \\ \begin{array}{c|cccc|} & A & B & C & D \\ A & 0 & 1 & 0 & 1 \\ B & 1 & 1 & 0 & 1 \\ C & 1 & 1 & 0 & 0 \\ D & 1 & 1 & 1 & 0 \end{array} \end{array} = V$$

verbindingsmatrix

16c Als je in een directe-wegenmatrix alle 'niet nullen' vervangt door 'enen' krijg je de verbindingsmatrix.
Omgekeerd kan niet, want aan een verbinding kun je niet het aantal wegen aflezen.

17a

$$M_1 + M_2 = M_{\text{totaal}} \Rightarrow \begin{array}{c|cccc|} & A & B & C & D \\ \text{fil 1} & 10 & 22 & 18 & 14 \\ \text{fil 2} & 17 & 18 & 12 & 8 \\ \text{fil 3} & 18 & 15 & 16 & 12 \end{array} + \begin{array}{c|cccc|} & A & B & C & D \\ \text{fil 1} & 30 & 18 & 10 & 15 \\ \text{fil 2} & 16 & 29 & 17 & 13 \\ \text{fil 3} & 18 & 21 & 16 & 13 \end{array} = \begin{array}{c|cccc|c} & A & B & C & D & \\ \text{fil 1} & 40 & 40 & 28 & 29 & 137 \\ \text{fil 2} & 33 & 47 & 29 & 21 & 130 \\ \text{fil 3} & 36 & 36 & 32 & 25 & 129 \\ \hline & 109 & 123 & 89 & 75 & \end{array}$$

17b $28+29+32=89$. Het totaal aan verkochte wasautomaten C in de drie filialen samen in de eerste helft van 2007.

17c $40+40+28+29=137$. Het totaal aantal verkochte wasautomaten in filiaal 1 in de eerste helft van 2007.

18a A is 2×2-matrix C is 4×1-matrix E is 2×4-matrix
B is 2×3-matrix D is 3×3-matrix F is 1×3-matrix.

18b A en D zijn vierkante matrices.
De som van de getallen op de hoofddiagonaal bij matrix A is $3+0=3$ en bij matrix D is dat $3+2+5=10$.

18c $a_{21}=1$ c_{14} bestaat niet $e_{24}=6$
$b_{21}=4$ $d_{31}=6$ $f_{12}=1$.

19a

$$\begin{array}{c|cccc|} & \text{L} & \text{LX} & \text{SLX} & \text{D} \\ \text{Nob.} & 4 & 6 & 3 & 5 \\ \text{Cas.} & 0 & 8 & 4 & 2 \end{array} = V_1$$

19b

$$\begin{array}{c|cccc|} & \text{L} & \text{LX} & \text{SLX} & \text{D} \\ \text{Nob.} & 1 & 5 & 8 & 3 \\ \text{Cas.} & 0 & 4 & 8 & 3 \end{array} = V_2$$

19c

$$\begin{array}{c|cccc|} & \text{L} & \text{LX} & \text{SLX} & \text{D} \\ \text{Nob.} & 5 & 11 & 11 & 8 \\ \text{Cas.} & 0 & 12 & 12 & 5 \end{array} = V$$

19d

$$\begin{array}{c|cccc|} & \text{L} & \text{LX} & \text{SLX} & \text{D} \\ \text{mei} & 4 & 14 & 7 & 7 \\ \text{juni} & 1 & 9 & 16 & 6 \end{array} = T$$

19e $14+9=23$ geeft het totale aantal auto's in de uitvoering LX dat in mei en juni door beide autobedrijven samen is verkocht.

20a

$$P = \begin{array}{c|c|} Q & 0{,}8 \\ T & 1{,}2 \\ L & 1{,}6 \\ S & 2{,}0 \end{array} \ (\times €1000).$$

20b Zaak A ontvangt $8\times 0{,}8+6\times 1{,}2+4\times 1{,}6+3\times 2{,}0=26$ ($\times$ €1000).
Zaak B ontvangt $6\times 0{,}8+3\times 1{,}2+0\times 1{,}6+5\times 2{,}0=18{,}4$ ($\times$ €1000).

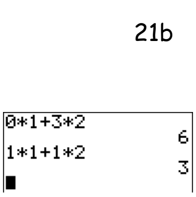
20c

$$I = \begin{array}{c|c|} A & 26 \\ B & 18{,}4 \end{array} \ (\times €1000).$$

21a

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} = B$$

$$A = \begin{pmatrix} 0 & 3 \\ 1 & 1 \end{pmatrix} \qquad \begin{pmatrix} 6 \\ 3 \end{pmatrix} = A \cdot B$$



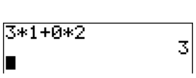
```
0*1+3*2
                6
1*1+1*2
                3
▮
```

21b Kan niet.

21c

$$\begin{pmatrix} 1 \\ 2 \end{pmatrix} = B$$

$$C = \begin{pmatrix} 3 & 0 \end{pmatrix} \qquad (3) = C \cdot B$$

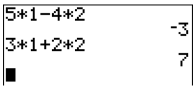


```
3*1+0*2
                3
▮
```

21d

$$\begin{pmatrix}1\\2\end{pmatrix}=B$$

$$D=\begin{pmatrix}5&-4\\3&2\end{pmatrix}\quad\begin{pmatrix}-3\\7\end{pmatrix}=D\cdot B$$

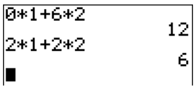

```
5*1-4*2
                  -3
3*1+2*2
                   7
■
```

21e $A\cdot B+D\cdot B=\begin{pmatrix}6\\3\end{pmatrix}+\begin{pmatrix}-3\\7\end{pmatrix}=\begin{pmatrix}3\\10\end{pmatrix}.$

21f

$$\begin{pmatrix}1\\2\end{pmatrix}=B$$

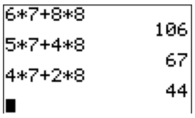
$$2A=\begin{pmatrix}0&6\\2&2\end{pmatrix}\quad\begin{pmatrix}12\\6\end{pmatrix}=2A\cdot B$$


```
0*1+6*2
                  12
2*1+2*2
                   6
■
```

22a $T=\begin{matrix}A\\B\end{matrix}\begin{pmatrix}7\\8\end{pmatrix}.$

22b

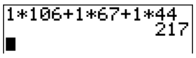
$$\begin{array}{c|c}
 & \begin{matrix}A\\B\end{matrix}\begin{pmatrix}7\\8\end{pmatrix}\\
\hline
P=\begin{array}{c}\\3\\5\\10\end{array}\begin{array}{c}\begin{matrix}A&B\end{matrix}\\\begin{pmatrix}6&8\\5&4\\4&2\end{pmatrix}\end{array} & \begin{matrix}3\\5\\10\end{matrix}\begin{pmatrix}106\\67\\44\end{pmatrix}=P\cdot T=V
\end{array}$$


```
6*7+8*8
                 106
5*7+4*8
                  67
4*7+2*8
                  44
■
```

De betekenis van de matrix hierboven:
het zijn de aantallen fietsen die per dag gemaakt worden met 3, 5 of 10 versnellingen.

22c

$$\begin{array}{c|c}
 & \begin{matrix}3\\5\\10\end{matrix}\begin{pmatrix}106\\67\\44\end{pmatrix}\\
\hline
W=\begin{pmatrix}1&1&1\end{pmatrix} & (217)=W\cdot V
\end{array}$$

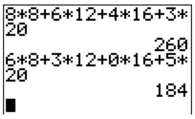

```
1*106+1*67+1*44
                 217
■
```

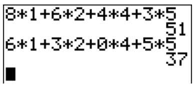
Betekenis:
het aantal fietsen dat per dag gemaakt wordt.

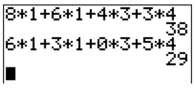
23ab 💻

$$\left.\begin{array}{c|ccc}
 & \text{prijs} & \text{winst} & \text{software}\\
Q & 8 & 1 & 1\\
T & 12 & 2 & 1\\
L & 16 & 4 & 3\\
S & 20 & 5 & 4
\end{array}\right)(\times €100)=W$$

$$V=\begin{array}{c|cccc}
 & Q & T & L & S\\
A & 8 & 6 & 4 & 3\\
B & 6 & 3 & 0 & 5
\end{array}\qquad
\begin{array}{c|ccc}
 & \text{prijs} & \text{winst} & \text{software}\\
A & 260 & 51 & 38\\
B & 184 & 37 & 29
\end{array}(\times €100)=K$$


```
8*8+6*12+4*16+3*
20
                 260
6*8+3*12+0*16+5*
20
                 184
■
```

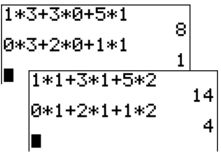

```
8*1+6*2+4*4+3*5
                  51
6*1+3*2+0*4+5*5
                  37
■
```


```
8*1+6*1+4*3+3*4
                  38
6*1+3*1+0*3+5*4
                  29
■
```

24a 💻

$$\begin{pmatrix}3&1\\0&1\\1&2\end{pmatrix}=Q$$

$$P=\begin{pmatrix}1&3&5\\0&2&1\end{pmatrix}\quad\begin{pmatrix}8&14\\1&4\end{pmatrix}=P\cdot Q$$


```
1*3+3*0+5*1
                   8
0*3+2*0+1*1
                   1
■
```
```
1*1+3*1+5*2
                  14
0*1+2*1+1*2
                   4
■
```

24b 💻

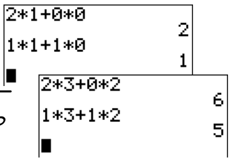
$$\begin{pmatrix}2&0\\1&1\end{pmatrix}=R$$

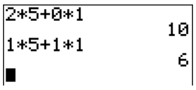
$P=\begin{pmatrix}1&3&5\\0&2&1\end{pmatrix}$ $P\cdot R$ kan niet

24c 💻

$$\begin{pmatrix}1&3&5\\0&2&1\end{pmatrix}=P$$

$$R=\begin{pmatrix}2&0\\1&1\end{pmatrix}\quad\begin{pmatrix}2&6&10\\1&5&6\end{pmatrix}=R\cdot P$$

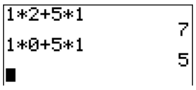

```
2*1+0*0
                   2
1*1+1*0
                   1
■
```
```
2*3+0*2
                   6
1*3+1*2
                   5
■
```


```
2*5+0*1
                  10
1*5+1*1
                   6
■
```

24d 💻

$$\begin{pmatrix}2&0\\1&1\end{pmatrix}=R$$

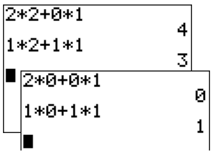
$$S=\begin{pmatrix}1&5\end{pmatrix}\quad\begin{pmatrix}7&5\end{pmatrix}=S\cdot R$$


```
1*2+5*1
                   7
1*0+5*1
                   5
■
```

24e 💻

$$\begin{pmatrix}2&0\\1&1\end{pmatrix}=R$$

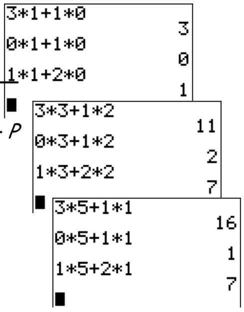
$$R=\begin{pmatrix}2&0\\1&1\end{pmatrix}\quad\begin{pmatrix}4&0\\3&1\end{pmatrix}=R\cdot R$$


```
2*2+0*1
                   4
1*2+1*1
                   3
■
```
```
2*0+0*1
                   0
1*0+1*1
                   1
■
```

24f 💻

$$\begin{pmatrix}1&3&5\\0&2&1\end{pmatrix}=P$$

$$Q=\begin{pmatrix}3&1\\0&1\\1&2\end{pmatrix}\quad\begin{pmatrix}3&11&16\\0&2&1\\1&7&7\end{pmatrix}=Q\cdot P$$


```
3*1+1*0
                   3
0*1+1*0
                   0
1*1+2*0
                   1
■
```
```
3*3+1*2
                  11
0*3+1*2
                   2
1*3+2*2
                   7
■
```
```
3*5+1*1
                  16
0*5+1*1
                   1
1*5+2*1
                   7
■
```

25ab 💻

$$\left.\begin{array}{c|cc}
 & \text{inkoop} & \text{verkoop}\\
A & 400 & 500\\
B & 275 & 400\\
C & 600 & 800\\
D & 900 & 1200
\end{array}\right)=P$$

$$V=\begin{array}{c|cccc}
 & A & B & C & D\\
\text{filiaal 1} & 10 & 12 & 8 & 4\\
\text{filiaal 2} & 7 & 8 & 5 & 2\\
\text{filiaal 3} & 8 & 8 & 6 & 2
\end{array}\qquad
\begin{array}{c|cc}
 & \text{inkoop} & \text{verkoop}\\
\text{filiaal 1} & 15700 & 21000\\
\text{filiaal 2} & 9800 & 13100\\
\text{filiaal 3} & 10800 & 14400
\end{array}=V\cdot P=T$$

t_{21} geeft het totale inkoopbedrag van filiaal 2.

25cd 💻

$$\left.\begin{array}{c|cc}
 & \text{inkoop} & \text{verkoop}\\
\text{filiaal 1} & 15700 & 21000\\
\text{filiaal 2} & 9800 & 13100\\
\text{filiaal 3} & 10800 & 14400
\end{array}\right)=T$$

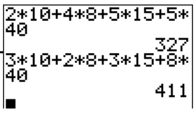
$$Q=\begin{array}{ccc}\text{f1}&\text{f2}&\text{f3}\\(1&1&1)\end{array}\qquad\begin{array}{cc}\text{inkoop}&\text{verkoop}\\(36300&48500)\end{array}=Q\cdot T$$

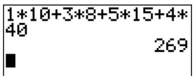
De totale winst is $48\,500-36\,300=12\,200$ (€).

26ac

$$\left.\begin{array}{c|c}
 & \text{kosten}\\
P & 10\\
Q & 8\\
R & 15\\
T & 40
\end{array}\right)=K$$

$$M=\begin{array}{c|cccc}
 & P & Q & R & T\\
A & 2 & 4 & 5 & 5\\
B & 3 & 2 & 3 & 8\\
C & 1 & 3 & 5 & 4
\end{array}\qquad
\begin{array}{c|c}
 & \text{kosten}\\
A & 327\\
B & 411\\
C & 269
\end{array}=M\cdot K$$


```
2*10+4*8+5*15+5*
40
                 327
3*10+2*8+3*15+8*
40
                 411
■
```


```
1*10+3*8+5*15+4*
40
                 269
■
```

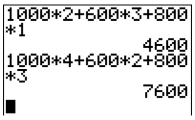
$M\cdot K$ geeft de productiekosten van A, B en C per eenheid.

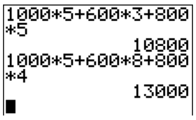
26bd

$$\left.\begin{array}{c|cccc}
 & P & Q & R & T\\
A & 2 & 4 & 5 & 5\\
B & 3 & 2 & 3 & 8\\
C & 1 & 3 & 5 & 4
\end{array}\right)=M$$

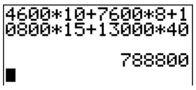
$$N=\text{best.}\begin{array}{ccc}A&B&C\\(1000&600&800)\end{array}\qquad\text{best.}\begin{array}{cccc}P&Q&R&T\\(4600&7600&10800&13000)\end{array}=N\cdot M$$

$N\cdot M$ geeft de benodigde hoeveelheid grondstoffen en arbeidstijd die nodig is voor de bestelling.


```
1000*2+600*3+800
*1
                4600
1000*4+600*2+800
*3
                7600
■
```


```
1000*5+600*3+800
*5
               10800
1000*5+600*8+800
*4
               13000
■
```

26e
$$(N\cdot M)\cdot K=\begin{array}{cc} & \begin{array}{cccc}P&Q&R&T\end{array}\\ \text{best.}&\begin{pmatrix}4600&7600&10800&13000\end{pmatrix}\end{array}\cdot\begin{array}{cc} & \text{kosten}\\ \begin{matrix}P\\Q\\R\\T\end{matrix}&\begin{pmatrix}10\\8\\15\\40\end{pmatrix}\end{array}=\begin{array}{cc} & \text{kosten}\\ \text{best.}&\begin{pmatrix}788800\end{pmatrix}\end{array}.$$



```
4600*10+7600*8+1
0800*15+13000*40
                  788800
█
```

De productiekosten van de totale bestelling zijn 788800 (€).

27a
$$A=\begin{array}{cc} & 7\\ \begin{matrix}4\\5\\6\\7\\8\end{matrix}&\begin{pmatrix}0\\0\\0\\1\\0\end{pmatrix}\end{array}.$$

27b
$$B=\begin{array}{cc} & \text{vold.}\\ \begin{matrix}4\\5\\6\\7\\8\end{matrix}&\begin{pmatrix}0\\0\\1\\1\\1\end{pmatrix}\end{array}.$$

27c
$$C=\begin{array}{cc} & \text{tot.}\\ \begin{matrix}4\\5\\6\\7\\8\end{matrix}&\begin{pmatrix}1\\1\\1\\1\\1\end{pmatrix}\end{array}.$$

27d
$$D=\begin{array}{cc} & \text{pnt.}\\ \begin{matrix}4\\5\\6\\7\\8\end{matrix}&\begin{pmatrix}4\\5\\6\\7\\8\end{pmatrix}\end{array}.$$

27e
$$E=\begin{array}{cc} & \begin{array}{ccc}5A1&5A2&5A3\end{array}\\ \text{aant.}&\begin{pmatrix}1&1&1\end{pmatrix}\end{array}.$$

28a
$$\begin{array}{c|c} & \begin{pmatrix}a&b&c\\d&e&f\\g&h&i\end{pmatrix}=M\\ \hline P=\begin{pmatrix}3&0&0\\0&3&0\\0&0&3\end{pmatrix}&\begin{pmatrix}3a&3b&3c\\3d&3e&3f\\3g&3h&3i\end{pmatrix}=P\cdot M=3M\end{array}$$

28b
$$\begin{array}{c|c} & \begin{pmatrix}a&b&c\\d&e&f\\g&h&i\end{pmatrix}=M\\ \hline Q=\begin{pmatrix}1&0&0\\0&3&0\\0&0&0\end{pmatrix}&\begin{pmatrix}a&b&c\\3d&3e&3f\\0&0&0\end{pmatrix}=Q\cdot M\end{array}$$

28c
$$\begin{array}{c|c} & \begin{pmatrix}1&0&0\\1&0&1\\0&2&0\end{pmatrix}=R\\ \hline M=\begin{pmatrix}a&b&c\\d&e&f\\g&h&i\end{pmatrix}&\begin{pmatrix}a+b&2c&b\\d+e&2f&e\\g+h&2i&h\end{pmatrix}=M\cdot R\end{array}$$

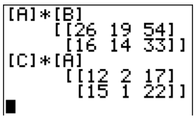
29a
$$\begin{array}{c|c} & \begin{pmatrix}1&0&0&0&0\\0&1&0&0&0\\0&0&1&0&0\\0&0&0&1&0\\0&0&0&0&1\end{pmatrix}=A\\ \hline M=\begin{pmatrix}a&b&c&d&e\\f&g&h&i&j\end{pmatrix}&\begin{pmatrix}a&b&c&d&e\\f&g&h&i&j\end{pmatrix}=M\cdot A=M\end{array}$$

29b
$$\begin{array}{c|c} & \begin{pmatrix}a&b&c&d&e\\f&g&h&i&j\end{pmatrix}=M\\ \hline B=\begin{pmatrix}1&0\\0&1\end{pmatrix}&\begin{pmatrix}a&b&c&d&e\\f&g&h&i&j\end{pmatrix}=B\cdot M=M\end{array}$$

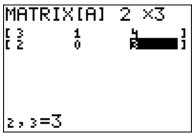
✲✲✲ **🖩 Neem GR - practicum 13 door.** (zie aan het eind van deze uitwerkingen)

30a $P\cdot Q=\begin{pmatrix}26&19&54\\16&14&33\end{pmatrix}$.

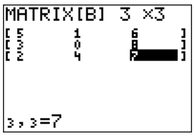
30b $R\cdot P=\begin{pmatrix}12&2&17\\15&1&22\end{pmatrix}$.



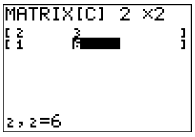
```
[A]*[B]
     [[26 19 54]
      [16 14 33]]
[C]*[A]
      [[12 2 17]
       [15 1 22]]
█
```



```
MATRIX[A] 2 ×3
[ 3      1      4      ]
[ 2      0      3      ]

2,3=3
```

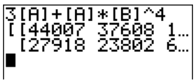


```
MATRIX[B] 3 ×3
[ 5      1      6      ]
[ 3      0      8      ]
[ 2      4      7      ]

3,3=7
```

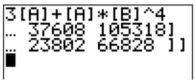


```
MATRIX[C] 2 ×2
[ 2      3             ]
[ 1      6             ]

2,2=6
```

30c $3P+P\cdot Q^4=\begin{pmatrix}44007&37608&105318\\27918&23802&66828\end{pmatrix}$.



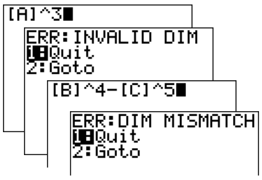
```
3[A]+[A]*[B]^4
[[44007 37608 1…
 [27918 23802 6…
█
```



```
3[A]+[A]*[B]^4
… 37608 105318]
… 23802 66828 ]]
█
```

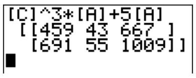
30d P^3 geeft een foutmelding, want P is geen vierkante matrix.

30e $Q^4-R^5=$ geeft een foutmelding want Q^4 en R^5 hebben niet dezelfde afmeting.



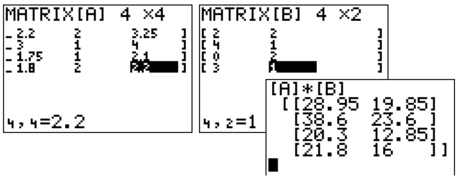
```
[A]^3█
ERR:INVALID DIM
1:Quit
2:Goto
[B]^4-[C]^5█
ERR:DIM MISMATCH
1:Quit
2:Goto
```

30f $R^3\cdot P+5P=\begin{pmatrix}459&43&667\\691&55&1009\end{pmatrix}$.



```
[C]^3*[A]+5[A]
 [[459 43 667 ]
  [691 55 1009]]
█
```

31a
$$T\cdot G=\begin{array}{cc} & \begin{array}{cccc}\text{volw.}&\text{kind}&\text{dier}&\text{tent}\end{array}\\ \begin{matrix}\text{camping 1}\\\text{camping 2}\\\text{camping 3}\\\text{camping 4}\end{matrix}&\begin{pmatrix}5{,}20&2{,}20&2{,}00&3{,}25\\7{,}30&3{,}00&1{,}00&4{,}00\\3{,}50&1{,}75&1{,}00&2{,}10\\4{,}00&1{,}80&2{,}00&2{,}20\end{pmatrix}\end{array}\cdot\begin{array}{cc} & \begin{array}{cc}V&E\end{array}\\ \begin{matrix}\text{volw.}\\\text{kind}\\\text{dier}\\\text{tent}\end{matrix}&\begin{pmatrix}2&2\\4&1\\0&2\\3&1\end{pmatrix}\end{array}=\begin{array}{cc} & \begin{array}{cc}V&E\end{array}\\ \begin{matrix}\text{camping 1}\\\text{camping 2}\\\text{camping 3}\\\text{camping 4}\end{matrix}&\begin{pmatrix}28{,}95&19{,}85\\38{,}60&23{,}60\\20{,}30&12{,}85\\21{,}80&16\end{pmatrix}\end{array}=K.$$



```
MATRIX[A] 4 ×4
_2.2     2      3.25   ]
_3       1      4      ]
_1.75    1      2.1    ]
_1.8     2      2.2    ]

4,4=2.2
```

```
MATRIX[B] 4 ×2
[ 2      2             ]
[ 4      1             ]
[ 0      2             ]
[ 3      1             ]

4,2=1
```

```
[A]*[B]
 [[28.95 19.85]
  [38.6  23.6 ]
  [20.3  12.85]
  [21.8  16   ]]
█
```

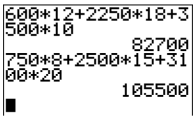
31b $k_{21}=38{,}60$ is het bedrag dat de familie Vrielink (met 2 volw, 4 kinderen en 3 tenten) per dag betaalt op camping 2.

31c Camping 3 was zowel voor familie Vrielink als voor familie Eijssink het voordeligst.

31d $\begin{pmatrix}7&7&7&7\end{pmatrix}\cdot K=F$ geeft voor beide families de bedragen als ze op elke camping één week hebben gestaan.

32a De elementen e_{12} en e_{21} van $P\cdot V$ hebben geen betekenis.
(je vermenigvuldigt dan de voorraad van het ene merk met de prijzen van het andere merk)

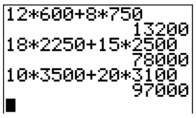
32b
$$P\cdot V=\begin{array}{cc} & \begin{array}{ccc}\text{breedb.}&\text{plasma}&\text{led}\end{array}\\ \begin{matrix}A\\B\end{matrix}&\begin{pmatrix}600&2250&3500\\750&2500&3100\end{pmatrix}\end{array}\cdot\begin{array}{cc} & \begin{array}{cc}A&B\end{array}\\ \begin{matrix}\text{breedb.}\\\text{plasma}\\\text{led}\end{matrix}&\begin{pmatrix}12&8\\18&15\\10&20\end{pmatrix}\end{array}=\begin{array}{cc} & \begin{array}{cc}A&B\end{array}\\ \begin{matrix}A\\B\end{matrix}&\begin{pmatrix}82700&\ldots\\\ldots&105500\end{pmatrix}\end{array}.$$



```
600*12+2250*18+3
500*10
                   82700
750*8+2500*15+31
00*20
                  105500
█
```

e_{11} is de waarde (€) van de TV's van merk A die in voorraad zijn.
e_{22} is de waarde (€) van de TV's van merk B die in voorraad zijn.

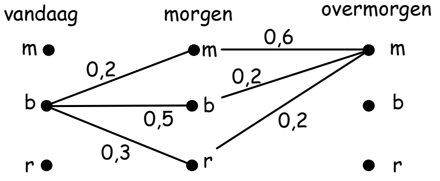
32c
$$V\cdot P=\begin{array}{r}\\ \text{breedb.}\\ \text{plasma}\\ \text{led}\end{array}\!\!\begin{array}{c}\begin{array}{cc}A & B\end{array}\\ \begin{pmatrix}12 & 8\\ 18 & 15\\ 10 & 20\end{pmatrix}\end{array}\cdot\begin{array}{r}\\ A\\ B\end{array}\!\!\begin{array}{c}\begin{array}{ccc}\text{breedb.} & \text{plasma} & \text{led}\end{array}\\ \begin{pmatrix}600 & 2250 & 3500\\ 750 & 2500 & 3100\end{pmatrix}\end{array}=\begin{array}{r}\\ \text{breedb.}\\ \text{plasma}\\ \text{led}\end{array}\!\!\begin{array}{c}\begin{array}{ccc}\text{breedb.} & \text{plasma} & \text{led}\end{array}\\ \begin{pmatrix}13200 & \ldots & \ldots\\ \ldots & 78000 & \ldots\\ \ldots & \ldots & 97000\end{pmatrix}\end{array}.$$



```
12*600+8*750
           13200
18*2250+15*2500
           78000
10*3500+20*3100
           97000
```

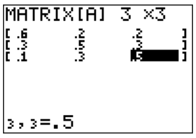
Alleen de elementen op de hoofddiagonaal van $V\cdot P$ hebben betekenis. (de opbrengst bij verkoop van de totale voorraad)
(bij de andere elementen vermenigvuldig je bijvoorbeeld de prijs van breedbeeld A met de voorraad van plasma A)

33 P(overmorgen mooi weer onder de voorwaarde dat het vandaag bewolkt is)
= P(overmorgen mooi weer| vandaag bewolkt)
= P(morgen mooi weer en overmorgen mooi weer| vandaag bewolkt)
+ P(morgen bewolkt en overmorgen mooi weer| vandaag bewolkt)
+ P(morgen regen en overmorgen mooi weer| vandaag bewolkt)
$= 0{,}2\cdot 0{,}6+0{,}5\cdot 0{,}2+0{,}3\cdot 0{,}2=0{,}12+0{,}10+0{,}06=0{,}28.$

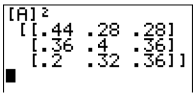


34a De som van de getallen in een kolom is de som van de kansen op alle mogelijke volgende weertypen.
De kansen in een rij hebben niets met elkaar te maken.

34b
$$W^2=\text{naar}\begin{array}{c}\\ \text{m}\\ \text{b}\\ \text{r}\end{array}\!\!\begin{array}{c}\text{van}\\ \begin{array}{ccc}\text{m} & \text{b} & \text{r}\end{array}\\ \begin{pmatrix}0{,}44 & 0{,}28 & 0{,}28\\ 0{,}36 & 0{,}40 & 0{,}36\\ 0{,}20 & 0{,}32 & 0{,}36\end{pmatrix}\end{array}.$$



```
MATRIX[A] 3 ×3
[ .6      .2      .2     ]
[ .3      .5      .3     ]
[ .1      .3      .5     ]

3,3=.5
```

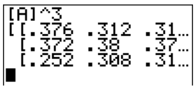


```
[A]²
 [[.44 .28 .28]
  [.36 .4  .36]
  [.2  .32 .36]]
```

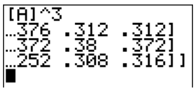
34c 0,20 van W^2 is de kans dat als het op een dag in mei mooi weer is, het twee dagen later zal regenen.

34d Dit is element w_{13} van W^2. Deze kans is 0,28.

34e
$$W^3=\text{naar}\begin{array}{c}\\ \text{m}\\ \text{b}\\ \text{r}\end{array}\!\!\begin{array}{c}\text{van}\\ \begin{array}{ccc}\text{m} & \text{b} & \text{r}\end{array}\\ \begin{pmatrix}0{,}376 & 0{,}312 & 0{,}312\\ 0{,}372 & 0{,}380 & 0{,}372\\ 0{,}252 & 0{,}308 & 0{,}316\end{pmatrix}\end{array}.$$



```
[A]^3
[[.376 .312 .31…
 [.372 .38  .37…
 [.252 .308 .31…
```



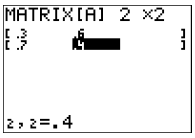
```
[A]^3
…376 .312 .312]
…372 .38  .372]
…252 .308 .316]]
```

0,372 van W^3 is de kans dat als het op een dag in mei regent, het drie dagen later bewolkt zal zijn.

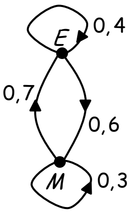
34f 6 mei is 3 dagen na 3 mei. De gevraagde kans wordt gegeven door element w_{33} van W^3 en is 0,316.

34g Voor Egypte zullen de getallen op de eerste rij vrijwel gelijk aan 1 zijn en de andere getallen vrijwel nul.

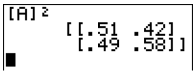
35a Zie de graaf hiernaast.
$$P=\text{naar}\begin{array}{c}\\ M\\ E\end{array}\!\!\begin{array}{c}\text{van}\\ \begin{array}{cc}M & E\end{array}\\ \begin{pmatrix}0{,}3 & 0{,}6\\ 0{,}7 & 0{,}4\end{pmatrix}\end{array}.$$



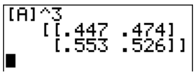
```
MATRIX[A] 2 ×2
[ .3      .6     ]
[ .7      .4     ]

2,2=.4
```



35b
$$P^2=\text{naar}\begin{array}{c}\\ M\\ E\end{array}\!\!\begin{array}{c}\text{van}\\ \begin{array}{cc}M & E\end{array}\\ \begin{pmatrix}0{,}51 & 0{,}42\\ 0{,}49 & 0{,}58\end{pmatrix}\end{array}\text{ en }P^3=\text{naar}\begin{array}{c}\\ M\\ E\end{array}\!\!\begin{array}{c}\text{van}\\ \begin{array}{cc}M & E\end{array}\\ \begin{pmatrix}0{,}447 & 0{,}474\\ 0{,}553 & 0{,}526\end{pmatrix}\end{array}.$$



```
[A]²
     [[.51 .42]
      [.49 .58]]
```

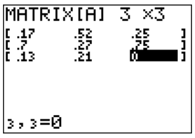


```
[A]^3
   [[.447 .474]
    [.553 .526]]
```

35c De kansen voor hoofdstuk 2 staan nu in P.
De kansen voor hoofdstuk 3 staan in P^2. De gevraagde kans (van M naar E) is 0,49.

35d De kansen voor h4 staan nu in P, die voor h5 in P^2 en voor h6 in P^3. De gevraagde kans (van E naar M) is 0,474.

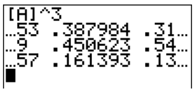
36a
$$\text{naar}\begin{array}{c}\\ \text{K}\\ \text{M}\\ \text{S}\end{array}\!\!\begin{array}{c}\text{van}\\ \begin{array}{ccc}\text{K} & \text{M} & \text{S}\end{array}\\ \begin{pmatrix}0{,}17 & 0{,}52 & 0{,}25\\ 0{,}70 & 0{,}27 & 0{,}75\\ 0{,}13 & 0{,}21 & 0\end{pmatrix}\end{array}=T.$$



```
MATRIX[A] 3 ×3
[ .17     .52     .25    ]
[ .7      .27     .75    ]
[ .13     .21     0      ]

3,3=0
```

0,27 is de kans dat na een medeklinker weer een medeklinker komt.

36b
$$\text{naar}\begin{array}{c}\\ \text{K}\\ \text{M}\\ \text{S}\end{array}\!\!\begin{array}{c}\text{van}\\ \begin{array}{ccc}\text{K} & \text{M} & \text{S}\end{array}\\ \begin{pmatrix}0{,}325 & 0{,}388 & 0{,}317\\ 0{,}534 & 0{,}451 & 0{,}547\\ 0{,}140 & 0{,}161 & 0{,}136\end{pmatrix}\end{array}=T^3.$$

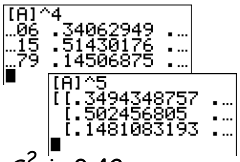


```
[A]^3
…53 .387984 .31…
…9  .450623 .54…
…57 .161393 .13…
```

0,451 is de kans dat drie plaatsen na een medeklinker weer een medeklinker komt.

36c De kans dat vier plaatsen na een medeklinker weer een medeklinker komt, is (ongeveer) 0,514.

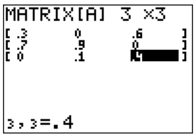
36d De kans dat vijf plaatsen na een klinker een spatie komt, is (ongeveer) 0,148.



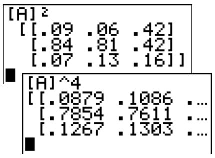
```
[A]^4
…06 .34062949 .…
…15 .51430176 .…
…79 .14506875 .…
```

```
[A]^5
[[.3494348757 .…
 [.502456805  .…
 [.1481083193 .…
```

37a
$$\text{naar}\begin{array}{c}\\ \text{B}\\ \text{M}\\ \text{E}\end{array}\!\!\begin{array}{c}\text{van}\\ \begin{array}{ccc}\text{B} & \text{M} & \text{E}\end{array}\\ \begin{pmatrix}0{,}3 & 0 & 0{,}6\\ 0{,}7 & 0{,}9 & 0\\ 0 & 0{,}1 & 0{,}4\end{pmatrix}\end{array}=S.$$



```
MATRIX[A] 3 ×3
[ .3      0       .6     ]
[ .7      .9      0      ]
[ 0       .1      .4     ]

3,3=.4
```



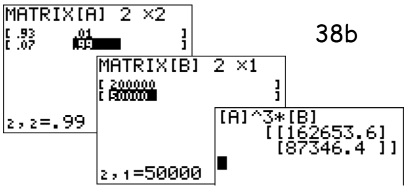
```
[A]²
 [[.09 .06 .42]
  [.84 .81 .42]
  [.07 .13 .16]]
```

```
[A]^4
[[.0879 .1086 .…
 [.7854 .7611 .…
 [.1267 .1303 .…
```

37b Element s_{13} van S^2 is 0,42.
De kans dat twee klanken na een eindklank een beginklank komt.

37c Element s_{12} van S^4 is 0,1086.

38a
$$\text{naar}\begin{array}{c}\\ \text{A}\\ \text{R}\end{array}\!\!\begin{array}{c}\text{van}\\ \begin{array}{cc}\text{A} & \text{R}\end{array}\\ \begin{pmatrix}0{,}93 & 0{,}01\\ 0{,}07 & 0{,}99\end{pmatrix}\end{array}=V.$$



```
MATRIX[A] 2 ×2
[ .93     .01    ]
[ .07     .99    ]

2,2=.99
```

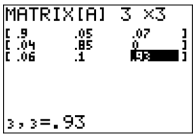
```
MATRIX[B] 2 ×1
[ 200000         ]
[ 50000          ]

2,1=50000
```

```
[A]^3*[B]
    [[162653.6]
     [87346.4 ]]
```

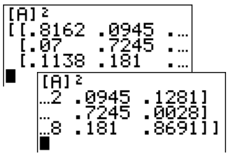
38b
$$\text{naar}\begin{array}{c}\\ \text{A}\\ \text{R}\end{array}\!\!\begin{array}{c}\text{van}\\ \begin{array}{cc}\text{A} & \text{R}\end{array}\\ \begin{pmatrix}0{,}93 & 0{,}01\\ 0{,}07 & 0{,}99\end{pmatrix}^3\end{array}\cdot\text{naar}\begin{array}{c}\\ \text{A}\\ \text{R}\end{array}\!\!\begin{array}{c}\text{van}\\ \text{inw}\\ \begin{pmatrix}200000\\ 50000\end{pmatrix}\end{array}\approx\text{naar}\begin{array}{c}\\ \text{A}\\ \text{R}\end{array}\!\!\begin{array}{c}\text{van}\\ \text{inw}\\ \begin{pmatrix}162650\\ 87350\end{pmatrix}\end{array}.$$

A heeft na 15 jaar 162650 inwoners en R 87350.

39a Bereken met de GR element w_{11} van W^2. Dat is 0,8162 ⇒ 81,6%.



```
MATRIX[A] 3 ×3
[ .9    .05   .07  ]
[ .04   .85   0    ]
[ .06   .1    .93  ]

3,3=.93
```

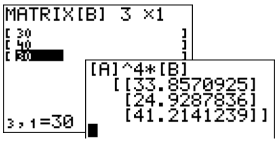


```
[A]²
[[.8162 .0945 .…
 [.07   .7245 .…
 [.1138 .181  .…
```

```
[A]²
…2 .0945 .1281]
…  .7245 .0028]
…8 .181  .8691]]
```

39b

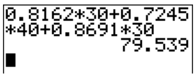
$$W^4 \cdot \text{naar} \begin{array}{cc} & \text{van} \\ & \% \\ A & 30 \\ B & 40 \\ C & 30 \end{array} \approx \text{naar} \begin{array}{cc} & \text{van} \\ & \% \\ A & 33{,}9 \\ B & 24{,}9 \\ C & 41{,}2 \end{array}.$$



```
MATRIX[B] 3 ×1
[ 30 ]
[ 40 ]
[ 30 ]

3,1=30
```

```
[A]^4*[B]
 [[33.8570925]
  [24.9287836]
  [41.2141239]]
```

39c Neem de elementen w_{11}, w_{22} en w_{33} van W^2.
Je krijgt $0{,}8162 \cdot 30 + 0{,}7245 \cdot 40 + 0{,}8291 \cdot = 79{,}539$. Dus 79,5%.



```
0.8162*30+0.7245
*40+0.8691*30
          79.539
```

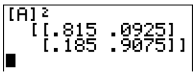
39d De elementen w_{31}, w_{32} en w_{33} van W worden groter.
(de kolomsommen moeten wel 1 blijven)
Zie bijvoorbeeld de matrix W hiernaast.

$$\text{naar} \begin{array}{cccc} & & \text{van} & \\ & A & B & C \\ A & 0{,}87 & 0{,}04 & 0{,}05 \\ B & 0{,}03 & 0{,}81 & 0 \\ C & 0{,}10 & 0{,}15 & 0{,}95 \end{array} = W$$

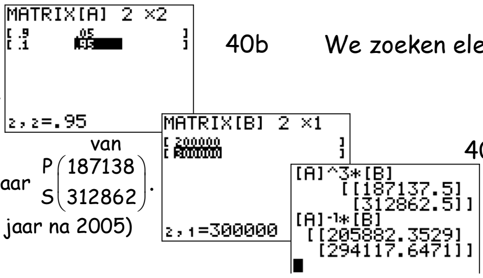
40a

$$\text{naar} \begin{array}{ccc} & \text{van} & \\ & P & S \\ P & 0{,}90 & 0{,}05 \\ S & 0{,}10 & 0{,}95 \end{array} = T.$$

40b We zoeken element t_{22} van T^2 ⇒ 0,9075.



```
[A]²
 [[.815 .0925]
  [.185 .9075]]
```



```
MATRIX[A] 2 ×2
[ .9   .05 ]
[ .1   .95 ]

2,2=.95
```

```
MATRIX[B] 2 ×1
[ 200000 ]
[ 300000 ]

2,1=300000
```

```
[A]^3*[B]
   [[187137.5]
    [312862.5]]
[A]⁻¹*[B]
 [[205882.3529]
  [294117.6471]]
```

40c

$$T^3 \cdot \text{naar} \begin{array}{cc} & \text{van} \\ P & 200000 \\ S & 300000 \end{array} = \text{naar} \begin{array}{cc} & \text{van} \\ P & 187138 \\ S & 312862 \end{array}.$$

(2020 is 3 perioden van 5 jaar na 2005)

40d

$$T^{-1} \cdot \text{naar} \begin{array}{cc} & \text{van} \\ P & 200000 \\ S & 300000 \end{array} = \text{naar} \begin{array}{cc} & \text{van} \\ P & 205882 \\ S & 294118 \end{array}.$$

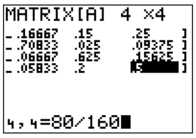
Dus in 2000 had de stad 294120 inwoners.

41a Deze kans is $\frac{55}{80}$.

41b

$$\text{naar} \begin{array}{ccccc} & & \text{van} & & \\ & N & F & S & E \\ N & \frac{55}{80} & \frac{20}{120} & \frac{6}{40} & \frac{40}{160} \\ F & \frac{10}{80} & \frac{85}{120} & \frac{1}{40} & \frac{15}{160} \\ S & \frac{5}{80} & \frac{8}{120} & \frac{25}{40} & \frac{25}{160} \\ E & \frac{10}{80} & \frac{7}{120} & \frac{8}{40} & \frac{80}{160} \end{array} \approx \text{naar} \begin{array}{ccccc} & & \text{van} & & \\ & N & F & S & E \\ N & 0{,}688 & 0{,}167 & 0{,}150 & 0{,}250 \\ F & 0{,}125 & 0{,}708 & 0{,}025 & 0{,}094 \\ S & 0{,}062 & 0{,}067 & 0{,}625 & 0{,}156 \\ E & 0{,}125 & 0{,}058 & 0{,}200 & 0{,}500 \end{array} = V.$$

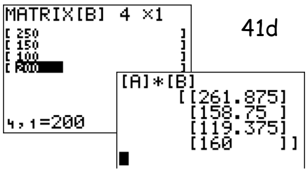
De kolomsommen moeten 1 zijn.
(op de GR niet afronden)



```
MATRIX[A] 4 ×4
_ .16667  .15    .25     ]
_ .70833  .025   .09375  ]
_ .06667  .625   .15625  ]
_ .05833  .2     .5      ]

4,4=80/160
```

41c

$$V \cdot \text{naar} \begin{array}{cc} & \text{van} \\ & \text{totaal} \\ N & 250 \\ F & 150 \\ S & 100 \\ E & 200 \end{array} \approx \text{naar} \begin{array}{cc} & \text{van} \\ & \text{totaal} \\ N & 262 \\ F & 159 \\ S & 119 \\ E & 160 \end{array}.$$



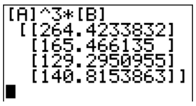
```
MATRIX[B] 4 ×1
[ 250 ]
[ 150 ]
[ 100 ]
[ 200 ]

4,1=200
```

```
[A]*[B]
     [[261.875]
      [158.75 ]
      [119.375]
      [160    ]]
```

De komende zomer zullen 262 werknemers in Nederland blijven en er zullen 159 werknemers naar Frankrijk gaan.

41d

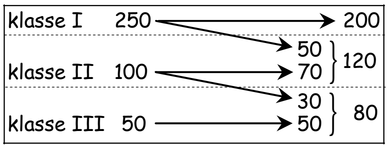
$$V^3 \cdot \text{naar} \begin{array}{cc} & \text{van} \\ & \text{totaal} \\ N & 250 \\ F & 150 \\ S & 100 \\ E & 200 \end{array} \approx \text{naar} \begin{array}{cc} & \text{van} \\ & \text{totaal} \\ N & 264 \\ F & 166 \\ S & 129 \\ E & 161 \end{array}.$$



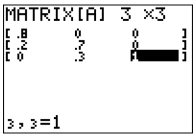
```
[A]^3*[B]
 [[264.4233832]
  [165.466135 ]
  [129.2950955]
  [140.8153863]]
```

Er zullen (over 3 jaar) 129 werknemers naar Spanje gaan.

42a $P(\text{boom blijft in I}) = \frac{200}{250} = 0{,}8$.
$P(\text{boom van I naar II}) = \frac{50}{250} = 0{,}2$.
De tabel geeft overgangen hiernaast.



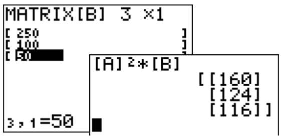
$$\text{naar} \begin{array}{cccc} & & \text{van} & \\ & I & II & III \\ I & 0{,}8 & 0 & 0 \\ II & 0{,}2 & 0{,}7 & 0 \\ III & 0 & 0{,}3 & 1 \end{array} = V$$



```
MATRIX[A] 3 ×3
[ .8   0    0  ]
[ .2   .7   0  ]
[ 0    .3   1  ]

3,3=1
```

42b

$$V^2 \cdot \text{naar} \begin{array}{cc} & \text{van} \\ & \text{totaal} \\ I & 250 \\ II & 100 \\ III & 50 \end{array} = \text{naar} \begin{array}{cc} & \text{van} \\ & \text{totaal} \\ I & 160 \\ II & 124 \\ III & 116 \end{array}.$$

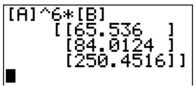


```
MATRIX[B] 3 ×1
[ 250 ]
[ 100 ]
[ 50  ]

3,1=50
```

```
[A]²*[B]
      [[160]
       [124]
       [116]]
```

42c

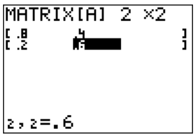
$$V^6 \cdot \text{naar} \begin{array}{cc} & \text{van} \\ & \text{totaal} \\ I & 250 \\ II & 100 \\ III & 50 \end{array} \approx \text{naar} \begin{array}{cc} & \text{van} \\ & \text{totaal} \\ I & 66 \\ II & 84 \\ III & 250 \end{array}.$$



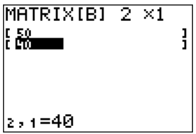
```
[A]^6*[B]
    [[65.536  ]
     [84.0124 ]
     [250.4516]]
```

43a

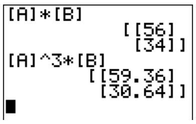
$$\text{naar} \begin{array}{ccc} & \text{van} & \\ & A & B \\ A & 0{,}8 & 0{,}4 \\ B & 0{,}2 & 0{,}6 \end{array} = M.$$



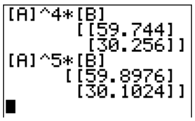
```
MATRIX[A] 2 ×2
[ .8   .4 ]
[ .2   .6 ]

2,2=.6
```



```
MATRIX[B] 2 ×1
[ 50 ]
[ 40 ]

2,1=40
```



```
[A]*[B]
      [[56]
       [34]]
[A]^3*[B]
   [[59.36]
    [30.64]]
```



```
[A]^4*[B]
    [[59.744]
     [30.256]]
[A]^5*[B]
   [[59.8976]
    [30.1024]]
```

43b De tweede week:

$$M \cdot \text{naar} \begin{array}{cc} & \text{van} \\ & \text{totaal} \\ A & 50 \\ B & 40 \end{array} = \text{naar} \begin{array}{cc} & \text{van} \\ & \text{totaal} \\ A & 56 \\ B & 34 \end{array};$$

De vierde week:

$$M^3 \cdot \text{naar} \begin{array}{cc} & \text{van} \\ & \text{totaal} \\ A & 50 \\ B & 40 \end{array} \approx \text{naar} \begin{array}{cc} & \text{van} \\ & \text{totaal} \\ A & 59 \\ B & 31 \end{array};$$

De vijfde week:

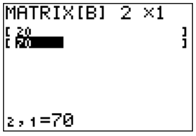
$$M^4 \cdot \text{naar} \begin{array}{cc} & \text{van} \\ & \text{totaal} \\ A & 50 \\ B & 40 \end{array} \approx \text{naar} \begin{array}{cc} & \text{van} \\ & \text{totaal} \\ A & 60 \\ B & 30 \end{array};$$

De zesde week:

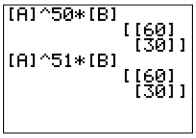
$$M^5 \cdot \text{naar} \begin{array}{cc} & \text{van} \\ & \text{totaal} \\ A & 50 \\ B & 40 \end{array} \approx \text{naar} \begin{array}{cc} & \text{van} \\ & \text{totaal} \\ A & 60 \\ B & 30 \end{array}.$$

43c Op den duur zullen de aantallen stabiliseren op 60 mappen A en 30 mappen B.

43d Ook nu op den duur 60 mappen A en 30 mappen B. (zie de GR-schermen hiernaast)



```
MATRIX[B] 2 ×1
[ 20 ]
[ 70 ]

2,1=70
```

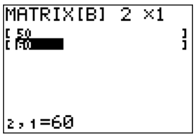


```
[A]^50*[B]
      [[60]
       [30]]
[A]^51*[B]
      [[60]
       [30]]
```

44a

$$M = \text{naar} \begin{array}{ccc} & \text{van} & \\ & L & S \\ L & 0{,}8 & 0{,}3 \\ S & 0{,}2 & 0{,}7 \end{array} \text{ en } B = \text{naar} \begin{array}{cc} & \text{van} \\ & \text{totaal} \\ L & 50 \\ S & 60 \end{array}.$$

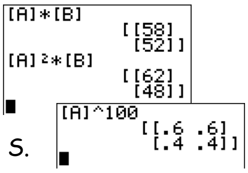
```
MATRIX[A] 2 ×2
[ .8   .3 ]
[ .2   .7 ]

2,2=.7
```



```
MATRIX[B] 2 ×1
[ 50 ]
[ 60 ]

2,1=60
```

44a (vervolg)

Over 5 jaar: $M\cdot B=\text{naar}\ \begin{array}{c@{}c} & \text{van totaal}\\ \begin{matrix}L\\S\end{matrix} & \begin{pmatrix}58\\52\end{pmatrix}\end{array}$ en over 10 jaar: $M^2\cdot B=\text{naar}\ \begin{array}{c@{}c} & \text{van totaal}\\ \begin{matrix}L\\S\end{matrix} & \begin{pmatrix}62\\48\end{pmatrix}\end{array}$.



```
[A]*[B]
             [[58]
              [52]]
[A]²*[B]
             [[62]
              [48]]
```

```
[A]^100
          [[.6 .6]
           [.4 .4]]
```

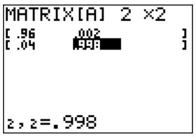
44b Op den duur 60% bij L en 40% bij S. (zie de GR-schermen hiernaast)
Dus op den duur $0{,}6\cdot110=66$ leden bij L en $0{,}4\cdot110=44$ leden bij S.

45ab

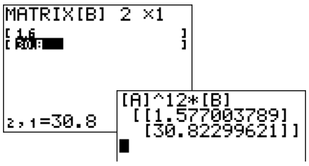
$M=\text{naar}\ \begin{array}{c@{}c} & \begin{matrix}\text{N} & \text{B}\end{matrix}\\ \begin{matrix}\text{N}\\\text{B}\end{matrix} & \begin{pmatrix}0{,}96 & 0{,}002\\0{,}04 & 0{,}998\end{pmatrix}\end{array}$ (van) en $K=\text{naar}\ \begin{array}{c@{}c} & \text{van totaal}\\ \begin{matrix}\text{N}\\\text{B}\end{matrix} & \begin{pmatrix}1{,}6\\30{,}8\end{pmatrix}\end{array}$.

Bij 1 januari 2003 (12 maanden verder) hoort $M^{12}\cdot B\approx\begin{array}{c@{}c}\begin{matrix}\text{N}\\\text{B}\end{matrix} & \begin{pmatrix}1{,}577\\30{,}823\end{pmatrix}\end{array}$.

Op 1 januari 2003 waren er 1,577 miljard munten in Nederland en 30,823 miljard in het buitenland.



```
MATRIX[A] 2 ×2
[ .96     .002     ]
[ .04     .998     ]

2,2=.998
```

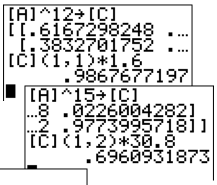


```
MATRIX[B] 2 ×1
[ 1.6              ]
[ 30.8             ]

2,1=30.8
```

```
[A]^12*[B]
 [[1.577003789]
  [30.82299621]]
```

45c Bij 1 januari 2003 hoort M^{12} met "van N(ederlandse munten) naar N(ederland)" $m_{11}\approx0{,}6167$.
Op 1 januari 2003 waren er $m_{11}\cdot1{,}6\approx0{,}987$ miljard Nederlandse munten in Nederland.

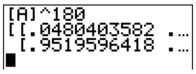
45d Bij 1 april 2003 hoort M^{15} met "van B(uitenlandse munten) naar N(ederland)" $m_{12}\approx0{,}0226$.
Op 1 april 2003 waren er $m_{12}\cdot30{,}8\approx0{,}696$ miljard buitenlandse munten in Nederland.



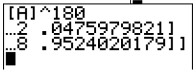
```
[A]^12→[C]
[[.6167298248 :…
 [.3832701752 :…
[C](1,1)*1.6
      .9867677197
```

```
[A]^15→[C]
…8 .0226004282]
…2 .9773995718]]
[C](1,2)*30.8
      .6960931873
```

45e Vanaf ongeveer M^{180} veranderen de machten van M niet meer.
Dus na ongeveer 180 maanden is de evenwichtssituatie bereikt.



```
[A]^180
[[.0480403582 :…
 [.9519596418 :…
```

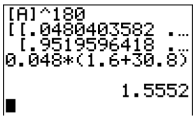


```
[A]^180
…2 .0475979821]
…8 .9524020179]]
```

45f

$M^{180}\approx\text{naar}\ \begin{array}{c@{}c} & \begin{matrix}\text{N} & \text{B}\end{matrix}\\ \begin{matrix}\text{N}\\\text{B}\end{matrix} & \begin{pmatrix}0{,}048 & 0{,}048\\0{,}952 & 0{,}952\end{pmatrix}\end{array}$ (van).

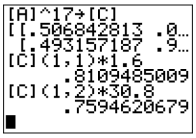
Op den duur zijn er $0{,}048\cdot(1{,}6+30{,}8)\approx1{,}555$ miljard munten in Nederland in omloop.
Hiervan heeft 4,8% de beeltenis van koningin Beatrix.



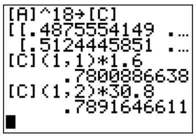
```
[A]^180
[[.0480403582 :…
 [.9519596418 :…
0.048*(1.6+30.8)
           1.5552
```

45g De vraag is: voor welke n in M^n is $m_{11}\cdot1{,}6=m_{12}\cdot30{,}8$.
Proberen geeft:

$$\left.\begin{array}{l} n=17\ \text{(1 juni 2003)}\Rightarrow\begin{cases}m_{11}\cdot1{,}6=0{,}811\\ m_{12}\cdot30{,}8=0{,}759\end{cases}\\ n=18\ \text{(1 juli 2003)}\Rightarrow\begin{cases}m_{11}\cdot1{,}6=0{,}780\\ m_{12}\cdot30{,}8=0{,}789\end{cases}\end{array}\right\}\Rightarrow\text{in juni 2003.}$$

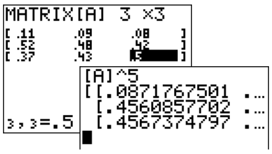


```
[A]^17→[C]
[[.506842813 .0…
 [.493157187 .9…
[C](1,1)*1.6
      .8109485009
[C](1,2)*30.8
      .7594620679
```



```
[A]^18→[C]
[[.4875554149 :…
 [.5124445851 :…
[C](1,1)*1.6
      .7800886638
[C](1,2)*30.8
      .7891646611
```

46a Bereken g_{11} van $G^5\Rightarrow g_{11}\approx0{,}087$. Dus 8,7% van de inactieven is vijf jaar later nog inactief.

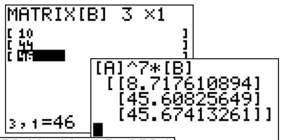


```
MATRIX[A] 3 ×3
[ .11     .09     .08   ]
[ .52     .48     .42   ]
[ .37     .43     .5    ]

3,3=.5
```

```
[A]^5
[[.0871767501 :…
 [.4560857702 :…
 [.4567374797 :…
```

46b

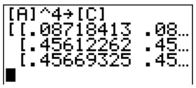
$G^7\cdot\text{naar}\ \begin{array}{c@{}c} & \begin{matrix}\text{I} & \text{M} & \text{N}\end{matrix}\\ \begin{matrix}\text{I}\\\text{M}\\\text{N}\end{matrix} & \begin{pmatrix}0{,}11 & 0{,}09 & 0{,}08\\0{,}52 & 0{,}48 & 0{,}42\\0{,}37 & 0{,}43 & 0{,}50\end{pmatrix}\end{array}\cdot\text{naar}\ \begin{array}{c@{}c} & \text{van }\%\\ \begin{matrix}\text{I}\\\text{M}\\\text{N}\end{matrix} & \begin{pmatrix}10\\44\\46\end{pmatrix}\end{array}\approx\text{naar}\ \begin{array}{c@{}c} & \text{van }\%\\ \begin{matrix}\text{I}\\\text{M}\\\text{N}\end{matrix} & \begin{pmatrix}8{,}7\\45{,}6\\45{,}7\end{pmatrix}\end{array}$. Dus 45,7%.



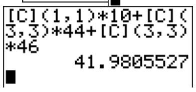
```
MATRIX[B] 3 ×1
[ 10             ]
[ 44             ]
[ 46             ]

3,1=46
```

```
[A]^7*[B]
 [[8.717610894]
  [45.60825649]
  [45.67413261]]
```

46c Bereken eerst g_{11}, g_{22} en g_{33}, van G^4.
Je vindt: $g_{11}\cdot10+g_{22}\cdot44+g_{33}\cdot46\approx42{,}0\%$.

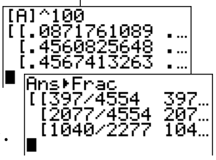


```
[A]^4→[C]
[[.087718413 .08…
 [.45612262  .45…
 [.45669325  .45…
```



```
[C](1,1)*10+[C](
3,3)*44+[C](3,3)
*46
       41.9805527
```

46d Op den duur zijn de drie kolommen van G^n gelijk $\Rightarrow$ stabilisatie.
Je vindt: $g_{11}=g_{12}=g_{12}=\frac{397}{4554}$; $g_{11}=g_{12}=g_{12}=\frac{2077}{4554}$; en $g_{31}=g_{32}=g_{32}=\frac{1040}{2277}$.

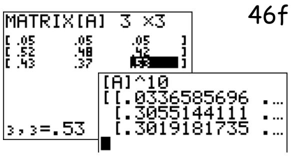


```
[A]^100
[[.0871761089 :…
 [.4560825648 :…
 [.4567413263 :…
```

```
Ans▸Frac
[[397/4554  397…
 [2077/4554 207…
 [1040/2277 104…
```

46e

$G=\text{naar}\ \begin{array}{c@{}c} & \begin{matrix}\text{I} & \text{M} & \text{N}\end{matrix}\\ \begin{matrix}\text{I}\\\text{M}\\\text{N}\end{matrix} & \begin{pmatrix}0{,}05 & 0{,}05 & 0{,}05\\0{,}52 & 0{,}48 & 0{,}42\\0{,}43 & 0{,}47 & 0{,}53\end{pmatrix}\end{array}$ (van).



```
MATRIX[A] 3 ×3
[ .05     .05     .05   ]
[ .52     .48     .42   ]
[ .43     .37     .53   ]

3,3=.53
```

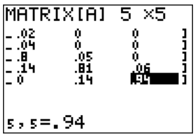
```
[A]^10
[[.0336585696 :…
 [.3055144111 :…
 [.3019181735 :…
```

46f Het lukt niet met de matrix van 46e.
Met de matrix hieraast lukt het natuurlijk al na 1 jaar.
(dus het is mogelijk)

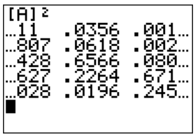
$G=\text{naar}\ \begin{array}{c@{}c} & \begin{matrix}\text{I} & \text{M} & \text{N}\end{matrix}\\ \begin{matrix}\text{I}\\\text{M}\\\text{N}\end{matrix} & \begin{pmatrix}0{,}06 & 0{,}06 & 0{,}06\\0{,}40 & 0{,}40 & 0{,}40\\0{,}54 & 0{,}54 & 0{,}54\end{pmatrix}\end{array}$ (van)

47a In elke kolom is de som van de elementen onder de hoofddiagonaal groter dan de som van de elementen erboven.

47b Bereken m_{33} van $M^2\Rightarrow m_{33}=0{,}6566$. Dus (ongeveer) 65,7%.



```
MATRIX[A] 5 ×5
….02     0       0     ]
….04     0       0     ]
….8      .05     0     ]
….14     .81     .06   ]
…0       .14     .94   ]

5,5=.94
```

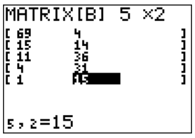


```
[A]²
…11  .0356 .001…
…807 .0618 .002…
…428 .6566 .080…
…627 .2264 .671…
…028 .0196 .245…
```

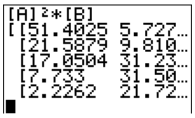
47c

$M^2\cdot\text{naar}\ \begin{array}{c@{}c} & \begin{matrix}\text{N} & A\ (\%)\end{matrix}\\ \begin{matrix}0\text{–}{<}5\\5\text{–}{<}10\\10\text{–}{<}25\\25\text{–}{<}50\\50+\end{matrix} & \begin{pmatrix}69 & 4\\15 & 14\\11 & 36\\4 & 31\\1 & 15\end{pmatrix}\end{array}=\text{naar}\ \begin{array}{c@{}c} & \begin{matrix}\text{N} & A\ (\%)\end{matrix}\\ \begin{matrix}0\text{–}{<}5\\5\text{–}{<}10\\10\text{–}{<}25\\25\text{–}{<}50\\50+\end{matrix} & \begin{pmatrix}51{,}4 & 5{,}7\\21{,}6 & 9{,}8\\17{,}1 & 31{,}2\\7{,}7 & 31{,}5\\2{,,}2 & 21{,}7\end{pmatrix}\end{array}$ (van).

Dus in Nederland (ongeveer) 7,7%
en in Amsterdam (ongeveer) 31,5%.



```
MATRIX[B] 5 ×2
[ 69      4        ]
[ 15      14       ]
[ 11      36       ]
[ 4       31       ]
[ 1       15       ]

5,2=15
```

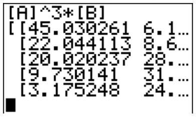


```
[A]²*[B]
[[51.4025  5.727…
 [21.5879  9.810…
 [17.0504  31.23…
 [7.733    31.50…
 [2.2262   21.72…
```

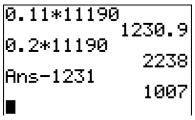
47d

$M^3\cdot\text{naar}\ \begin{array}{c@{}c} & \text{van N }(\%)\\ \begin{matrix}0\text{–}{<}5\\5\text{–}{<}10\\10\text{–}{<}25\\25\text{–}{<}50\\50+\end{matrix} & \begin{pmatrix}69\\15\\11\\4\\1\end{pmatrix}\end{array}=\text{naar}\ \begin{array}{c@{}c} & \text{van N }(\%)\\ \begin{matrix}0\text{–}{<}5\\5\text{–}{<}10\\10\text{–}{<}25\\25\text{–}{<}50\\50+\end{matrix} & \begin{pmatrix}45{,}0\\22{,}0\\20{,}0\\9{,}7\\3{,}2\end{pmatrix}\end{array}$.

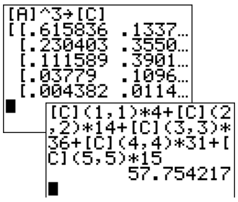
Dus $9{,}7+3{,}2=12{,}9\%$.



```
[A]^3*[B]
[[45.030261 6.1…
 [22.044113 8.6…
 [20.020237 28.…
 [9.730141  31.…
 [3.175248  24.…
```



```
0.11*11190
                1230.9
0.2*11190
                  2238
Ans-1231
                  1007
```



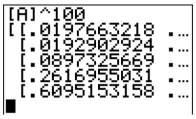
```
[A]^3→[C]
[[.615836 .1337…
 [.230403 .3550…
 [.111589 .3901…
 [.03779  .1096…
 [.004382 .0114…
```

```
[C](1,1)*4+[C](2
,2)*14+[C](3,3)*
36+[C](4,4)*31+[
C](5,5)*15
             57.754217
```

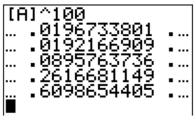
47e In 2004 in 11% van de buurten $\Rightarrow 0{,}11 \cdot 11190 \approx 1231$ buurten.
In 2019 in 20,0% (zie 47d) van de buurten $\Rightarrow 0{,}2 \cdot 11190 \approx 2238$ buurten.
Dat zijn $2238 - 1231 = 1007$ buurten meer.

47f Bereken eerst m_{11}, m_{22}, m_{33}, m_{44} en m_{55}, van M^3.
Je vindt: $m_{11} \cdot 4 + m_{22} \cdot 14 + m_{33} \cdot 36 + m_{44} \cdot 31 + m_{55} \cdot 15 \approx 57{,}8\%$ zit in dezelfde klasse.

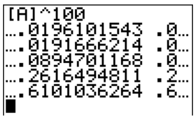
47g Voor grote n verandert M^n niet meer.
Dit betekent bijvoorbeeld dat op den duur in 26,2% van de buurten 25 tot 50 procent allochtoon is en dat in 61,0% van de buurten meer dan 50 procent allochtoon is.



```
[A]^100
[[.0197663218 .…
 [.0192902924 .…
 [.0897325669 .…
 [.2616955031 .…
 [.6095153158 .…
```



```
[A]^100
… .0196733801 .…
… .0192166909 .…
… .0895763736 .…
… .2616681149 .…
… .6098654405 .…
```



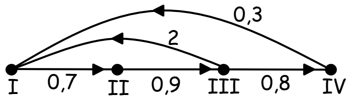
```
[A]^100
….0196101543 .0…
….0191666214 .0…
….0894701168 .0…
….2616494811 .2…
….6101036264 .6…
```

48a
- klasse IV is van 3 tot 4 jaar en er is geen klasse V.
- er loopt een pijl van klasse IV naar klasse I en hierbij staat de factor 1000.
- er loopt alleen een pijl van IV naar I en niet van I, II of III naar I.

48b Elke klasse duurt één jaar.

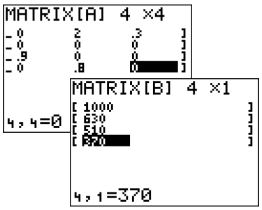
48c Op 1 juni 2007 in III: $0{,}6 \cdot 500 = 300$.
Op 1 juni 2008 in IV: $0{,}8 \cdot 300 = 240$.
Het jaar erna: $240 \cdot 1000 = 240\,000$ nakomelingen.

49a Zie de graaf hiernaast.



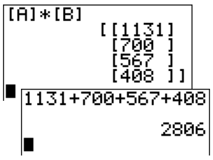
49b

$$L = \text{naar} \begin{array}{c} \\ \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \end{array} \begin{array}{c} \quad\text{van} \\ \begin{array}{cccc} \text{I} & \text{II} & \text{III} & \text{IV} \end{array} \\ \begin{pmatrix} 0 & 0 & 2 & 0{,}3 \\ 0{,}7 & 0 & 0 & 0 \\ 0 & 0{,}9 & 0 & 0 \\ 0 & 0 & 0{,}8 & 0 \end{pmatrix} \end{array}.$$



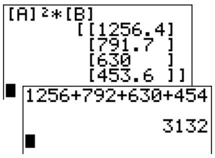
```
MATRIX[A] 4 ×4
[0    2    .3   ]
[0    0    0    ]
[.9   0    0    ]
[0    .8   0    ]

4,4=0
```

```
MATRIX[B] 4 ×1
[1000           ]
[630            ]
[510            ]
[370            ]

4,1=370
```



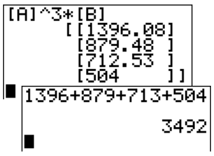
```
[A]*[B]
         [[1131]
          [700 ]
          [567 ]
          [408 ]]
```

```
1131+700+567+408
                2806
```



```
[A]²*[B]
       [[1256.4]
        [791.7 ]
        [630   ]
        [453.6 ]]
```

```
1256+792+630+454
                3132
```



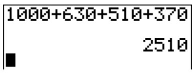
```
[A]^3*[B]
      [[1396.08]
       [879.48 ]
       [712.53 ]
       [504    ]]
```

```
1396+879+713+504
                3492
```

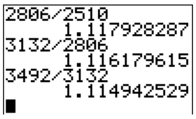
49cd

$t = 0$ $\quad P = \text{naar} \begin{array}{c} \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \end{array} \overset{\text{van}}{\begin{pmatrix} 1000 \\ 630 \\ 510 \\ 370 \end{pmatrix}}$. $\quad t = 1$ $\quad L \cdot P = \text{naar} \begin{array}{c} \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \end{array} \overset{\text{van}}{\begin{pmatrix} 1131 \\ 700 \\ 567 \\ 408 \end{pmatrix}}$. $\quad t = 2$ $\quad L^2 \cdot P \approx \text{naar} \begin{array}{c} \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \end{array} \overset{\text{van}}{\begin{pmatrix} 1256 \\ 792 \\ 630 \\ 454 \end{pmatrix}}$. $\quad t = 3$ $\quad L^3 \cdot P \approx \text{naar} \begin{array}{c} \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \end{array} \overset{\text{van}}{\begin{pmatrix} 1396 \\ 879 \\ 713 \\ 504 \end{pmatrix}}$.

$$2510 \xrightarrow{\times 1{,}118} 2806 \xrightarrow{\times 1{,}116} 3132 \xrightarrow{\times 1{,}115} 3492$$



```
1000+630+510+370
                2510
```

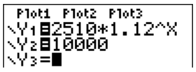


```
2806/2510
        1.117928287
3132/2806
        1.116179615
3492/3132
        1.114942529
```

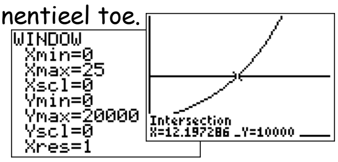
De groeifactoren zijn ongeveer gelijk $\Rightarrow$ de totale populatie neemt bij benadering exponentieel toe.

49e Exponentiële groei $\Rightarrow N = b \cdot g^t$ met $b = 2510$ en $g \approx 1{,}12 \Rightarrow N = 2510 \cdot 1{,}12^t$.

49f Los op: $2510 \cdot 1{,}12^t = 10\,000$ (intersect) $\Rightarrow t \approx 12{,}2$. Dus na ruim 12 jaar.



```
 Plot1  Plot2  Plot3
\Y1■2510*1.12^X
\Y2■10000
\Y3=
```



```
WINDOW
 Xmin=0
 Xmax=25
 Xscl=0
 Ymin=0
 Ymax=20000
 Yscl=0
 Xres=1
```

```
Intersection
X=12.197286  Y=10000
```

50a Er zijn 7 leeftijdsklassen $\Rightarrow$ de afmeting van de Lesliematrix L is 7×7.

50b Het element l_{21} (betekenis: de kans om van I naar II te gaan) in de Lesliematrix L is zeker niet nul.

50c l_{54} (betekenis: de kans om van IV naar V te gaan) in de Lesliematrix L is zeker niet nul.

50d l_{11} (betekenis: de kans om van I naar I te gaan) in de Lesliematrix L hoeft niet nul te zijn.
Het zou kunnen dat exemplaren uit I voor nakomelingen kunnen zorgen.
De andere elementen op de hoofddiagonaal moeten wel nul zijn omdat de exemplaren na een periode van 5 jaar naar de volgende klasse gaan.

50e De getallen in de eerste rij geven het gemideelde aantal nakomelingen per individu uit een leeftijdsklasse.

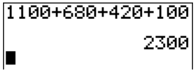
50f De andere getallen (die niet in de eerste rij staan) zijn kansen.

51a l_{32} is de kans om in een periode van 5 jaar uit de tweede in de derde leeftijdsklasse te komen.
l_{12} is het gemiddelde aantal nakomelingen in een periode van 5 jaar per individu uit de tweede leeftijdsklasse.
l_{21} is de kans om in een periode van 5 jaar uit de eerste in de tweede leeftijdsklasse te komen.
l_{11} is het gemiddelde aantal nakomelingen in een periode van 5 jaar per individu uit de eerste leeftijdsklasse.

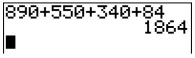
51b

$t = 0$ $\quad P = \text{naar} \begin{array}{c} \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \end{array} \overset{\text{van}}{\begin{pmatrix} 1100 \\ 680 \\ 420 \\ 100 \end{pmatrix}}$; $\quad t = 1$ $\quad L \cdot P = \text{naar} \begin{array}{c} \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \end{array} \overset{\text{van}}{\begin{pmatrix} 890 \\ 550 \\ 340 \\ 84 \end{pmatrix}}$; $\quad t = 2$ $\quad L^2 \cdot P = \text{naar} \begin{array}{c} \text{I} \\ \text{II} \\ \text{III} \\ \text{IV} \end{array} \overset{\text{van}}{\begin{pmatrix} 720 \\ 445 \\ 275 \\ 68 \end{pmatrix}}$.

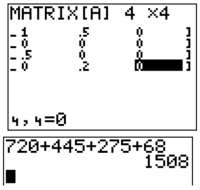
Totaal: 2300 1864 1508



```
1100+680+420+100
                2300
```

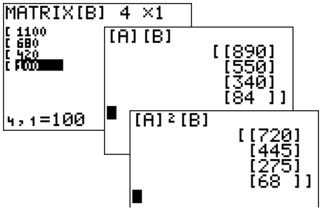


```
890+550+340+84
                1864
```



```
MATRIX[A] 4 ×4
[1    .5   0    ]
[0    0    0    ]
[.5   0    0    ]
[0    .2   0    ]

4,4=0
```

```
720+445+275+68
                1508
```



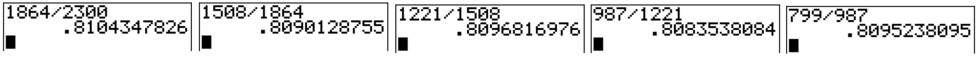
```
MATRIX[B] 4 ×1
[1100
[680
[420
[100

4,1=100
```

```
[A][B]
          [[890]
           [550]
           [340]
           [84 ]]
```

```
[A]²[B]
          [[720]
           [445]
           [275]
           [68 ]]
```

51c Totalen: in het begin: na 5 jaar: na10jaar: na 15 jaar: na 20 jaar: na 25 jaar:

$$2300 \xrightarrow{\times 0,81} 1864 \xrightarrow{\times 0,81} 1508 \xrightarrow{\times 0,81} 1221 \xrightarrow{\times 0,81} 987 \xrightarrow{\times 0,81} 799$$



```
1864/2300
      .8104347826
```

```
1508/1864
      .8090128755
```

```
1221/1508
      .8096816976
```

```
987/1221
      .8083538084
```

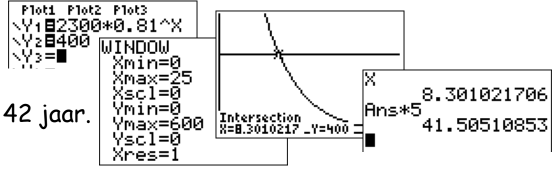
```
799/987
      .8095238095
```

De groeifactoren zijn ongeveer gelijk ⇒ de totale populatie neemt elke 5 jaar met ongeveer 19% af.

51d $N = 2300 \cdot 0,81^t$. (t gemeten in perioden van 5 jaar)

51e $N = 2300 \cdot 0,81^t = 400$ (intersect) $\Rightarrow t \approx 8,3$ (perioden van 5 jaar)

$N = 2300 \cdot 0,81^t < 400$ (zie een plot) ⇒ voor het eerst na (ongeveer) 42 jaar.



```
Plot1 Plot2 Plot3
\Y1=2300*0.81^X
\Y2=400
\Y3=
```

```
WINDOW
 Xmin=0
 Xmax=25
 Xscl=0
 Ymin=0
 Ymax=600
 Yscl=0
 Xres=1
```

```
Intersection
X=8.3010217 Y=400
```

```
X
          8.301021706
Ans*5
          41.50510853
```

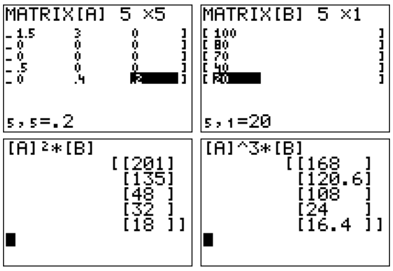
52a Omdat er 4-jarige (klasse ≥ 4) zijn die nog 5 jaar (ook klasse ≥ 4) worden is $l_{55} \neq 0$.
Er is gegeven dat 80% van klasse ≥ 4 binnen één jaar sterft $\Rightarrow l_{55} = 0,2$.
Haal verder uit beide bevolkingspiramiden:

$P(0 \to 1) = l_{21} = \frac{25+35}{48+52} = \frac{60}{100} = 0,6$; $P(1 \to 2) = l_{32} = \frac{30+34}{38+42} = \frac{64}{80} = 0,8$;

$P(2 \to 3) = l_{43} = \frac{17+18}{35+35} = \frac{35}{70} = 0,5$ en $P(3 \to 4) = l_{54} = \frac{10+10-0,2\cdot(9+11)}{17+23} = \frac{16}{40} = 0,4$.

Verder staat in de tekst: $P(2 \to 0) = l_{13} = 1,5$ en $P(3 \to 0) = l_{14} = 3$.
De andere elementen zijn 0. Hieruit volgt dan de Lesliematrix L hiernaast.

$$L = \text{naar}\ \begin{array}{c|ccccc} & & & \text{van} & & \\ & 0 & 1 & 2 & 3 & \geq 4 \\ \hline 0 & 0 & 0 & 1,5 & 3 & 0 \\ 1 & 0,6 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0,8 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0,5 & 0 & 0 \\ \geq 4 & 0 & 0 & 0 & 0,4 & 0,2 \end{array}$$



```
MATRIX[A] 5 ×5
 1.5      3      0    ]
 0        0      0    ]
 0        0      0    ]
 .5       0      0    ]
 0        .4     .2   ]

5,5=.2
```

```
MATRIX[B] 5 ×1
[ 100          ]
[ 80           ]
[ 70           ]
[ 40           ]
[ 20           ]

5,1=20
```

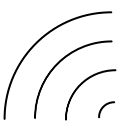
```
[A]²*[B]
          [[201]
           [135]
           [48 ]
           [32 ]
           [18 ]]
```

```
[A]^3*[B]
          [[168  ]
           [120.6]
           [108  ]
           [24   ]
           [16.4 ]]
```

52b

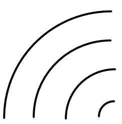
$$L^2 \cdot \text{naar}\ \begin{array}{c|c} & \text{van totaal} \\ \hline 0 & 100 \\ 1 & 80 \\ 2 & 70 \\ 3 & 40 \\ \geq 4 & 20 \end{array} \overset{t=2}{=} \text{naar}\ \begin{array}{c|c} & \text{van totaal} \\ \hline 0 & 201 \\ 1 & 135 \\ 2 & 48 \\ 3 & 32 \\ \geq 4 & 18 \end{array} \quad \text{en} \quad L^3 \cdot \text{naar}\ \begin{array}{c|c} & \text{van totaal} \\ \hline 0 & 100 \\ 1 & 80 \\ 2 & 70 \\ 3 & 40 \\ \geq 4 & 20 \end{array} \overset{t=3}{\approx} \text{naar}\ \begin{array}{c|c} & \text{van totaal} \\ \hline 0 & 168 \\ 1 & 121 \\ 2 & 108 \\ 3 & 24 \\ \geq 4 & 16 \end{array}.$$

53ab



$$\begin{array}{c|cccc} & \text{I} & \text{II} & \text{III} & \text{IV} \\ \hline \text{I} & 0 & 0 & 0 & d \\ \text{II} & a & 0 & 0 & 0 \\ \text{III} & 0 & b & 0 & 0 \\ \text{IV} & 0 & 0 & c & 0 \end{array} = L$$

$$L = \begin{array}{c|cccc} & \text{I} & \text{II} & \text{III} & \text{IV} \\ \hline \text{I} & 0 & 0 & 0 & d \\ \text{II} & a & 0 & 0 & 0 \\ \text{III} & 0 & b & 0 & 0 \\ \text{IV} & 0 & 0 & c & 0 \end{array} \quad \begin{array}{c|cccc} & \text{I} & \text{II} & \text{III} & \text{IV} \\ \hline \text{I} & 0 & 0 & cd & 0 \\ \text{II} & 0 & 0 & 0 & ad \\ \text{III} & ab & 0 & 0 & 0 \\ \text{IV} & 0 & bc & 0 & 0 \end{array} = L^2$$



$$\begin{array}{c|cccc} & \text{I} & \text{II} & \text{III} & \text{IV} \\ \hline \text{I} & 0 & 0 & cd & 0 \\ \text{II} & 0 & 0 & 0 & ad \\ \text{III} & ab & 0 & 0 & 0 \\ \text{IV} & 0 & bc & 0 & 0 \end{array} = L^2$$

$$L^2 = \begin{array}{c|cccc} & \text{I} & \text{II} & \text{III} & \text{IV} \\ \hline \text{I} & 0 & 0 & cd & 0 \\ \text{II} & 0 & 0 & 0 & ad \\ \text{III} & ab & 0 & 0 & 0 \\ \text{IV} & 0 & bc & 0 & 0 \end{array} \quad \begin{array}{c|cccc} & \text{I} & \text{II} & \text{III} & \text{IV} \\ \hline \text{I} & abcd & 0 & cd & 0 \\ \text{II} & 0 & abcd & 0 & 0 \\ \text{III} & 0 & 0 & abcd & 0 \\ \text{IV} & 0 & bc & 0 & abcd \end{array} = L^4$$

53b Je kunt in vier stappen alleen van een klasse naar diezelfde klasse. (bestudeer de graaf)

$\text{I} \xrightarrow{a} \text{II} \xrightarrow{b} \text{III} \xrightarrow{c} \text{V} \xrightarrow{d} \text{I} \Rightarrow \text{I} \xrightarrow[\text{in 4 stappen}]{abcd} \text{I} \Rightarrow$ in L^4 is $l_{11} = abcd$.

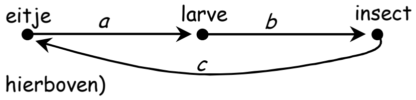
$\text{II} \xrightarrow{b} \text{III} \xrightarrow{c} \text{IV} \xrightarrow{d} \text{I} \xrightarrow{a} \text{II} \Rightarrow \text{I} \xrightarrow[\text{in 4 stappen}]{bcda} \text{I} \Rightarrow$ in L^4 is $l_{22} = abcd$.

$\text{III} \xrightarrow{c} \text{IV} \xrightarrow{d} \text{I} \xrightarrow{a} \text{II} \xrightarrow{b} \text{III} \Rightarrow \text{I} \xrightarrow[\text{in 4 stappen}]{cdab} \text{I} \Rightarrow$ in L^4 is $l_{33} = abcd$.

$\text{IV} \xrightarrow{d} \text{I} \xrightarrow{a} \text{II} \xrightarrow{b} \text{III} \xrightarrow{c} \text{IV} \Rightarrow \text{I} \xrightarrow[\text{in 4 stappen}]{dabc} \text{I} \Rightarrow$ in L^4 is $l_{44} = abcd$.

53c In elke klasse wordt het aantal dieren in vier jaar tijd vermenigvuldigd met $abcd$.

54a Maak eerst de graaf. (zie hierboven)



Maak met behulp van deze graaf de Lesliematrix L. (zie hiernaast)

$$L = \text{naar}\ \begin{array}{c|ccc} & & \text{van} & \\ & \text{e} & \text{l} & \text{i} \\ \hline \text{e} & 0 & 0 & c \\ \text{l} & a & 0 & 0 \\ \text{i} & 0 & b & 0 \end{array}$$

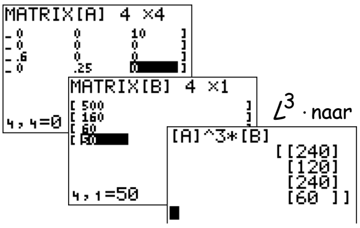
54b Maak met behulp van de driestapswegen in de graaf de matrix L^3. (zie hiernaast)

$$L^3 = \text{naar}\ \begin{array}{c|ccc} & & \text{van} & \\ & \text{e} & \text{l} & \text{i} \\ \hline \text{e} & abc & 0 & 0 \\ \text{l} & 0 & abc & 0 \\ \text{i} & 0 & 0 & abc \end{array}$$

54c $abc = 1$. (L^3 is de eenheidsmatrix) 54d $abc = 1,2$. 54e $abc < 1$.

55a P(pasgeboren dier wordt één jaar) $= 0,8 \cdot 0,6 = 0,48$.

55b Op 1 juli 2008, anderhalf jaar na 1 januari 2007, is $t = 3$.
De samenstelling op $t = 3$ bereken je met $L^3 \cdot P$.
(zie de berekening en het antwoord hiernaast)



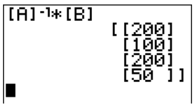
```
MATRIX[A] 4 ×4
 0        0      10   ]
 0        0      0    ]
 .6       0      0    ]
 0        .25    0    ]

4,4=0
```

```
MATRIX[B] 4 ×1
[ 500          ]
[ 160          ]
[ 60           ]
[ 50           ]

4,1=50
```

```
[A]^3*[B]
          [[240]
           [120]
           [240]
           [60 ]]
```

$$L^3 \cdot \text{naar}\ \begin{array}{c|c} & \text{van totaal} \\ \hline \text{I} & 500 \\ \text{II} & 160 \\ \text{III} & 60 \\ \text{IV} & 50 \end{array} \overset{t=3}{=} \text{naar}\ \begin{array}{c|c} & \text{van totaal} \\ \hline \text{I} & 240 \\ \text{II} & 120 \\ \text{III} & 240 \\ \text{IV} & 60 \end{array}$$

55c Op 1 juli 2006, een half jaar voor 1 januari 2007, is $t=-1$.
De samenstelling op $t=-1$ bereken je met $L^{-1}\cdot P$.
(zie de berekening en het antwoord hiernaast)

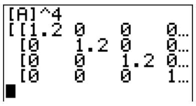


```
[A]⁻¹*[B]
          [[200]
           [100]
           [200]
           [50 ]]
█
```

$$L^{-1}\cdot\text{naar}\ \begin{array}{cc} & \text{van totaal}\\ \text{I} & 500\\ \text{II} & 160\\ \text{III} & 60\\ \text{IV} & 50 \end{array} = \text{naar}\ \begin{array}{cc} & \text{van totaal}\\ \text{I} & 200\\ \text{II} & 100\\ \text{III} & 200\\ \text{IV} & 50 \end{array}$$

$t=-1$

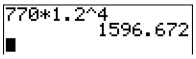
55d
$$L^4=\text{naar}\ \begin{array}{c|cccc} & \multicolumn{4}{c}{\text{van}}\\ & \text{I} & \text{II} & \text{III} & \text{IV}\\ \hline \text{I} & 1{,}2 & 0 & 0 & 0\\ \text{II} & 0 & 1{,}2 & 0 & 0\\ \text{III} & 0 & 0 & 1{,}2 & 0\\ \text{IV} & 0 & 0 & 0 & 1{,}2 \end{array}.$$



```
[A]^4
[[1.2 0   0   0…
 [0   1.2 0   0…
 [0   0   1.2 0…
 [0   0   0   1…
█
```

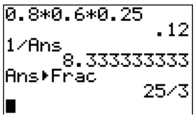
Dit betekent dat de hoeveelheid elke 2 jaar (vier halve jaren) met 20% toeneemt.

55e Elke twee jaar wordt de hoeveelheid met 1,20 vermenigvuldigd.
Op 1-1-2015 (4×2 jaar na 1-1-2007) zijn er $770\times1{,}20^4\approx1597$ dieren.



```
770*1.2^4
        1596.672
█
```

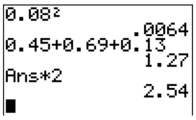
55f Over een periode van twee jaar (4 halve jaren) geen groei $\Rightarrow L^4$ is de eenheidsmatrix.
Er geldt dus $0{,}8\cdot0{,}6\cdot0{,}25\cdot v=1\Rightarrow0{,}12v=1\Rightarrow v=\frac{25}{3}\approx8{,}33$.
Vervang nu de 10 in de oorspronkelijk Lesliematrix door $\frac{25}{3}$. (afronden is niet nodig)



```
0.8*0.6*0.25
             .12
1/Ans
      8.333333333
Ans▸Frac
            25/3
█
```

56a P(twee perioden van 10 jaar 80+) $=0{,}080\cdot0{,}080=0{,}0064$.

56b Per individu $0{,}45+0{,}69+0{,}13=1{,}27$ kind $\Rightarrow$ per gezin $2\cdot1{,}27\approx2{,}5$ kind.

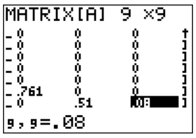


```
0.08²
           .0064
0.45+0.69+0.13
            1.27
Ans*2
            2.54
█
```

56c 2032 is 50 jaar (5 perioden van 10 jaar) na 1982 $\Rightarrow$ bereken $L^5\cdot P$.

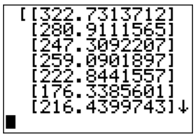
$$L=\text{naar}\ \begin{array}{c|ccccccccc} & \multicolumn{9}{c}{\text{van}}\\ & \text{0-9} & \text{10-19} & \text{20-29} & \text{30-39} & \text{40-49} & \text{50-59} & \text{60-69} & \text{70-79} & \text{80+}\\ \hline \text{0-9} & 0 & 0{,}45 & 0{,}69 & 0{,}13 & 0 & 0 & 0 & 0 & 0\\ \text{10-19} & 0{,}930 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ \text{20-29} & 0 & 0{,}993 & 0 & 0 & 0 & 0 & 0 & 0 & 0\\ \text{30-39} & 0 & 0 & 0{,}987 & 0 & 0 & 0 & 0 & 0 & 0\\ \text{40-49} & 0 & 0 & 0 & 0{,}981 & 0 & 0 & 0 & 0 & 0\\ \text{50-59} & 0 & 0 & 0 & 0 & 0{,}962 & 0 & 0 & 0 & 0\\ \text{60-69} & 0 & 0 & 0 & 0 & 0 & 0{,}907 & 0 & 0 & 0\\ \text{70-79} & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}761 & 0 & 0\\ \text{80+} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}510 & 0{,}080 \end{array} \Rightarrow L^5\cdot\text{naar}\ \begin{array}{cc} & \text{van totaal}\\ \text{0-9} & 205\\ \text{10-19} & 258\\ \text{20-29} & 169\\ \text{30-39} & 127\\ \text{40-49} & 99\\ \text{50-59} & 74\\ \text{60-69} & 48\\ \text{70-79} & 23\\ \text{80+} & 5\\ \hline & 1008 \end{array} \approx\text{naar}\ \begin{array}{cc} & \text{van totaal}\\ \text{0-9} & 323\\ \text{10-19} & 281\\ \text{20-29} & 247\\ \text{30-39} & 259\\ \text{40-49} & 223\\ \text{50-59} & 176\\ \text{60-69} & 216\\ \text{70-79} & 109\\ \text{80+} & 45\\ \hline & 1879 \end{array}.$$

1982 2032

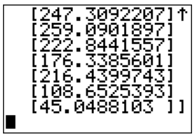


```
MATRIX[A] 9 ×9
 0     0     0   ↑
 0     0     0   ]
 0     0     0   ]
 0     0     0   ]
 0     0     0   ]
 .761  0     0   ]
 0     .51   .08 ]
9,9=.08
```

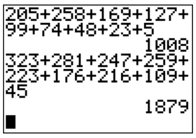
```
MATRIX[B] 9 ×1
[ 169          ↑
[ 127          ]
[ 99           ]
[ 74           ]
[ 48           ]
[ 23           ]
[ 5            ]
9,1=5
```



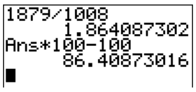
```
 [[322.7313712]
  [280.9111565]
  [247.3092207]
  [259.0901897]
  [222.8441557]
  [176.3385601]
  [216.4399743]↓
█
```



```
  [247.3092207]↑
  [259.0901897]
  [222.8441557]
  [176.3385601]
  [216.4399743]
  [108.6525393]
  [45.0488103 ]]
█
```



```
205+258+169+127+
99+74+48+23+5
            1008
323+281+247+259+
223+176+216+109+
45
            1879
█
```

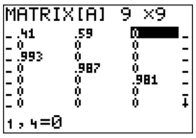


```
1879/1008
     1.864087302
Ans*100-100
     86.40873016
█
```

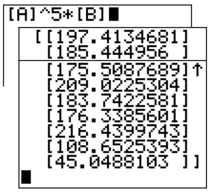
56d

2032

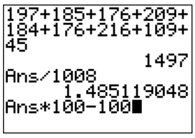
$$(L^*)^5\cdot P\approx\begin{pmatrix}197\\185\\176\\209\\184\\176\\216\\109\\45\end{pmatrix}.\qquad \begin{array}{c}\hline 1497\end{array}$$



```
MATRIX[A] 9 ×9
 .41   .59   0   
 0     0     0   
 .993  0     0   
 0     .987  0   
 0     0     .981
 0     0     0   
 0     0     0   ↓
1,4=0
```



```
[A]^5*[B]█
 [[197.4134681]
  [185.444956 ]
  [175.5087689]↑
  [209.0225304]
  [183.7422581]
  [176.3385601]
  [216.4399743]
  [108.6525393]
  [45.0488103 ]]
█
```



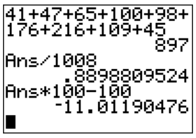
```
197+185+176+209+
184+176+216+109+
45
            1497
Ans/1008
     1.485119048
Ans*100-100█
```

De groei in 50 jaar is nu nog maar 48,6%.

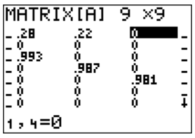
56e

2032

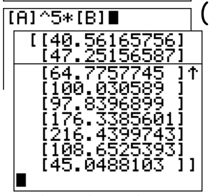
$$(L^{\sim})^5\cdot P\approx\begin{pmatrix}41\\47\\65\\100\\98\\176\\216\\109\\45\end{pmatrix}\qquad \begin{array}{c}\hline 897\end{array}$$



```
41+47+65+100+98+
176+216+109+45
             897
Ans/1008
     .8898809524
Ans*100-100
    -11.01190476
█
```



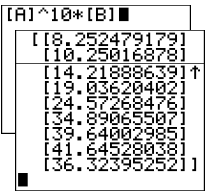
```
MATRIX[A] 9 ×9
 .28   .22   0   
 0     0     0   
 .993  0     0   
 0     .987  0   
 0     0     .981
 0     0     0   
 0     0     0   ↓
1,4=0
```



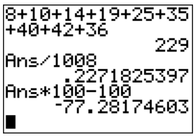
```
[A]^5*[B]█
 [[40.56165756]
  [47.25156587]
  [64.7757745 ]↑
  [100.030589 ]
  [97.8396899 ]
  [176.3385601]
  [216.4399743]
  [108.6525393]
  [45.0488103 ]]
█
```

2082

$$(L^{\sim})^{10}\cdot P\approx\begin{pmatrix}8\\10\\14\\19\\25\\35\\40\\42\\36\end{pmatrix}.\qquad \begin{array}{c}\hline 229\end{array}$$



```
[A]^10*[B]█
 [[8.252479179]
  [10.25016878]
  [14.21888639]↑
  [19.03620402]
  [24.57268476]
  [34.89065507]
  [39.64002985]
  [41.64528038]
  [36.32395252]]
█
```



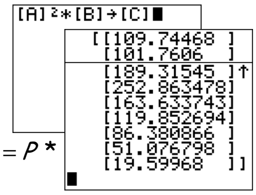
```
8+10+14+19+25+35
+40+42+36
             229
Ans/1008
     .2271825397
Ans*100-100
    -77.28174603
█
```

Op den duur loopt de bevolking dramatisch terug.

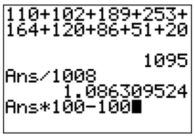
56f

2002

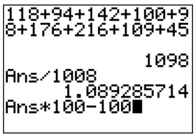
$$(L^{\sim})^2\cdot P\approx\begin{pmatrix}110\\102\\189\\253\\164\\120\\86\\51\\20\end{pmatrix}=P^*\qquad \begin{array}{c}\hline 1095\end{array}$$



```
[A]²*[B]→[C]█
 [[109.74468 ]
  [101.7606  ]
  [189.31545 ]↑
  [252.863478]
  [163.633743]
  [119.852694]
  [86.380866 ]
  [51.076798 ]
  [19.59968  ]]
█
```



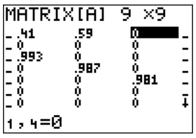
```
110+102+189+253+
164+120+86+51+20

            1095
Ans/1008
     1.086309524
Ans*100-100█
```



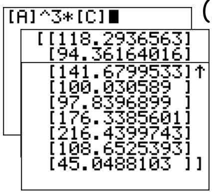
```
118+94+142+100+9
8+176+216+109+45

            1098
Ans/1008
     1.089285714
Ans*100-100█
```

2032

$$(L^*)^3\cdot P^*\approx\begin{pmatrix}118\\94\\142\\100\\98\\176\\216\\109\\45\end{pmatrix}\qquad \begin{array}{c}\hline 1098\end{array}$$



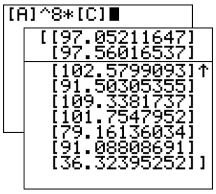
```
MATRIX[A] 9 ×9
 .41   .59   0   
 0     0     0   
 .993  0     0   
 0     .987  0   
 0     0     .981
 0     0     0   
 0     0     0   ↓
1,4=0
```



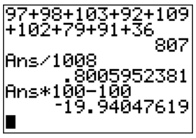
```
[A]^3*[C]█
 [[118.2936563]
  [94.36164016]
  [141.6799533]↑
  [100.030589 ]
  [97.8396899 ]
  [176.3385601]
  [216.4399743]
  [108.6525393]
  [45.0488103 ]]
```

2082

$$(L^*)^8\cdot P^*\approx\begin{pmatrix}97\\98\\103\\92\\109\\102\\79\\91\\36\end{pmatrix}\qquad \begin{array}{c}\hline 807\end{array}$$



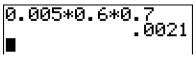
```
[A]^8*[C]█
 [[97.05211647]
  [97.56016537]
  [102.5799093]↑
  [91.50305355]
  [109.3381737]
  [101.7547952]
  [79.16136034]
  [91.08808691]
  [36.32395252]]
```



```
97+98+103+92+109
+102+79+91+36
             807
Ans/1008
     .8005952381
Ans*100-100
    -19.94047619
█
```

Op deze manier groeit de bevolking eerst bescheiden en daarna is er een afname.

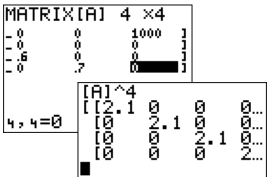
57a Er zijn 4 klassen van elk 2 jaar en $b_{44} = 0 \Rightarrow$ een baars wordt maximaal 8 jaar.
P(eitje groeit tot een baars van 6 jaar) $= 0{,}005 \cdot 0{,}6 \cdot 0{,}7 = 0{,}0021$.



```
0.005*0.6*0.7
            .0021
```

57b

$$B^4 = \text{naar}\ \begin{array}{c|cccc} & \multicolumn{4}{c}{\text{van}} \\ & \text{I} & \text{II} & \text{III} & \text{IV} \\ \hline \text{I} & 2{,}1 & 0 & 0 & 0 \\ \text{II} & 0 & 2{,}1 & 0 & 0 \\ \text{III} & 0 & 0 & 2{,}1 & 0 \\ \text{IV} & 0 & 0 & 0 & 2{,}1 \end{array} .$$

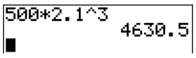


```
MATRIX[A] 4 ×4
[0    0    1000 ]
[0    0    0    ]
[.6   0    0    ]
[0    .7   0    ]
4,4=0
```

```
[A]^4
[[2.1  0    0    0…
 [0    2.1  0    0…
 [0    0    2.1  0…
 [0    0    0    2…
```

Elke 8 jaar (vier perioden van 2 jaar) wordt de populatie 2,1 ($= 1000 \cdot 0{,}005 \cdot 0{,}6 \cdot 0{,}7$) keer zo groot. Dus de populatie neemt in acht jaar met 110% toe.

57c Na 24 jaar (3 perioden van 8 jaar) zijn er $500 \cdot 2{,}1^3 \approx 4\,630$ baarzen. (afgerond op tientallen)

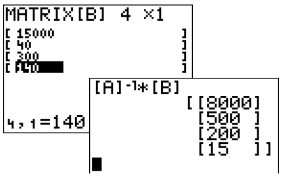


```
500*2.1^3
           4630.5
```

57d

1-1-2007 — 1-1-2005

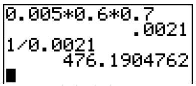
$$B^{-1} \cdot \text{naar}\ \begin{array}{c|c} & \text{van totaal} \\ \hline \text{I} & 15000 \\ \text{II} & 40 \\ \text{III} & 300 \\ \text{IV} & 140 \end{array} = \text{naar}\ \begin{array}{c|c} & \text{van totaal} \\ \hline \text{I} & 8000 \\ \text{II} & 500 \\ \text{III} & 200 \\ \text{IV} & 15 \end{array}$$



```
MATRIX[B] 4 ×1
[15000          ]
[40             ]
[300            ]
[140            ]
4,1=140
```

```
[A]-1*[B]
         [[8000]
          [500 ]
          [200 ]
          [15  ]]
```

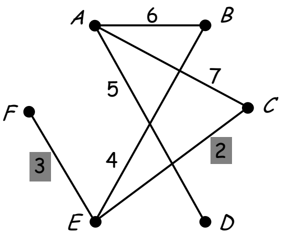
57e Een lagere vruchtbaarheid betekent dat b_{14} kleiner wordt.
Stel dat $b_{14} = v$ dan
$v \cdot 0{,}005 \cdot 0{,}6 \cdot 0{,}7 = 1$ (L^4 is de eenheidsmatrix)
$v \cdot 0{,}0021 = 1$
$v = \frac{1}{0{,}0021} \approx 476.$
Dus een baars legt dan gemiddeld 476 eitjes.



```
0.005*0.6*0.7
            .0021
1/0.0021
      476.1904762
```

Diagnostische toets

D1a Teken de zes punten A, B, C, D, E en F met de verbindingen die in V staan.
Zet de afstanden die uit M staan in de graaf.
$B \to F = B \to E \to F = 7 \Rightarrow E \to F = 3$.
$A \to E = A \to C \to E = 7 \Rightarrow C \to E = 2$.
Je krijgt dan de graaf hiernaast.
Langs alle verbindingen staan nu afstanden.
Matrix M is daarna geheel in te vullen.



$$\text{naar}\ \begin{array}{c|cccccc} & \multicolumn{6}{c}{\text{van}} \\ & A & B & C & D & E & F \\ \hline A & 0 & 6 & 7 & 5 & 9 & 12 \\ B & 6 & 0 & 6 & 11 & 4 & 7 \\ C & 7 & 6 & 0 & 12 & 2 & 5 \\ D & 5 & 11 & 12 & 0 & 14 & 17 \\ E & 9 & 4 & 2 & 14 & 0 & 3 \\ F & 12 & 7 & 5 & 17 & 3 & 0 \end{array} = M \qquad \begin{array}{c} \Sigma \\ 39 \\ 34 \\ 32 \\ 59 \\ 32 \\ 44 \end{array}$$

D1b Voor de af te leggen afstanden NAAR ... moet je de rijtotalen nemen.
Deze rijtotalenvan M zijn: 39, 34, 32, 59, 32 en 44.
De rijtotalen van C en E zijn het kleinst $\Rightarrow C$ en E genieten de voorkeur.

D1c De getallen in de eerste kolom moeten met 3 worden vermenigvuldigd, enz.
De rijtotalen worden: 69, 60, 62, 114, 60 en 75.
De rijtotalen van B en E zijn het kleinst $\Rightarrow B$ en E genieten de voorkeur.

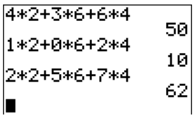
$$\text{naar}\ \begin{array}{c|cccccc} & \multicolumn{6}{c}{\text{van}} \\ & A & B & C & D & E & F \\ \hline A & 0 & 12 & 7 & 5 & 9 & 36 \\ B & 18 & 0 & 6 & 11 & 4 & 21 \\ C & 21 & 12 & 0 & 12 & 2 & 15 \\ D & 15 & 22 & 12 & 0 & 14 & 51 \\ E & 27 & 8 & 2 & 14 & 0 & 9 \\ F & 36 & 14 & 5 & 17 & 3 & 0 \end{array} = N \qquad \begin{array}{c} \Sigma \\ 69 \\ 60 \\ 62 \\ 114 \\ 60 \\ 75 \end{array}$$

D2a

$$\begin{pmatrix} 2 \\ 6 \\ 4 \end{pmatrix} = C$$

$$A = \begin{pmatrix} 4 & 3 & 6 \\ 1 & 0 & 2 \\ 2 & 5 & 7 \end{pmatrix} \quad \begin{pmatrix} 50 \\ 10 \\ 62 \end{pmatrix} = A \cdot C$$

mag ook met de optie matrix op de GR

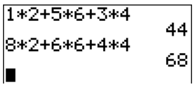


```
4*2+3*6+6*4
               50
1*2+0*6+2*4
               10
2*2+5*6+7*4
               62
```

D2b

$$\begin{pmatrix} 2 \\ 6 \\ 4 \end{pmatrix} = C$$

$$B = \begin{pmatrix} 1 & 5 & 3 \\ 8 & 6 & 4 \end{pmatrix} \quad \begin{pmatrix} 44 \\ 68 \end{pmatrix} = B \cdot C$$



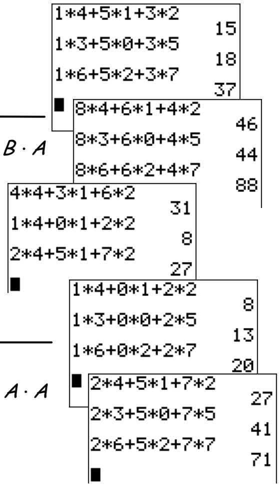
```
1*2+5*6+3*4
               44
8*2+6*6+4*4
               68
```

D2c $A \cdot B$ kan niet.

D2d

$$\begin{pmatrix} 4 & 3 & 6 \\ 1 & 0 & 2 \\ 2 & 5 & 7 \end{pmatrix} = A$$

$$B = \begin{pmatrix} 1 & 5 & 3 \\ 8 & 6 & 4 \end{pmatrix} \quad \begin{pmatrix} 15 & 18 & 37 \\ 46 & 44 & 88 \end{pmatrix} = B \cdot A$$



```
1*4+5*1+3*2
               15
1*3+5*0+3*5
               18
1*6+5*2+3*7
               37
```

```
8*4+6*1+4*2
               46
8*3+6*0+4*5
               44
8*6+6*2+4*7
               88
```

D2e $C \cdot D$ kan niet.

```
4*4+3*1+6*2
               31
1*4+0*1+2*2
                8
2*4+5*1+7*2
               27
```

D2f

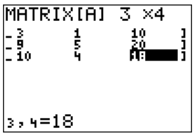
$$\begin{pmatrix} 4 & 3 & 6 \\ 1 & 0 & 2 \\ 2 & 5 & 7 \end{pmatrix} = A$$

$$A = \begin{pmatrix} 4 & 3 & 6 \\ 1 & 0 & 2 \\ 2 & 5 & 7 \end{pmatrix} \quad \begin{pmatrix} 31 & 42 & 72 \\ 8 & 13 & 20 \\ 27 & 41 & 71 \end{pmatrix} = A \cdot A$$

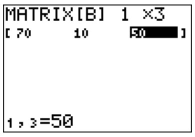
```
1*4+0*1+2*2
                8
1*3+0*0+2*5
               13
1*6+0*2+2*7
               20
```

```
2*4+5*1+7*2
               27
2*3+5*0+7*5
               41
2*6+5*2+7*7
               71
```

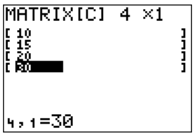
D3a $B = \text{eenh}\begin{array}{ccc} P_1 & P_2 & P_3 \\ (70 & 10 & 50) \end{array}$ en $C = \begin{array}{c|c} & € \\ G_1 & 10 \\ G_2 & 15 \\ G_3 & 20 \\ G_4 & 30 \end{array}$.



```
MATRIX[A] 3 ×4
[ 3    1    10  ]
[ 9    5    20  ]
[ 10   4    18  ]

3,4=18
```



```
MATRIX[B] 1 ×3
[ 70   10   50  ]

1,3=50
```

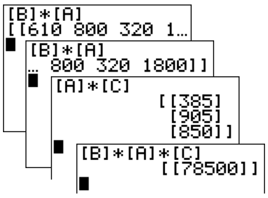


```
MATRIX[C] 4 ×1
[ 10  ]
[ 15  ]
[ 20  ]
[ 30  ]

4,1=30
```

D3b $B \cdot A = \text{eenh}\begin{array}{cccc} G_1 & G_2 & G_3 & G_4 \\ (610 & 800 & 320 & 1800) \end{array}$.
Voor de hele order zijn
610 eenheden G_1 nodig, enz.

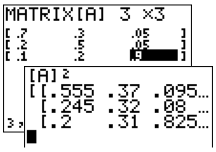
$A \cdot C = \begin{array}{c|c} & € \\ P_1 & 385 \\ P_2 & 905 \\ P_3 & 850 \end{array}$.
Het vervaardigen van
product P_1 kost € 385, enz.

$B \cdot A \cdot C = \text{eenh}\overset{€}{(78500)}$.
Voor de order is € 78 500
aan grondstofen nodig.



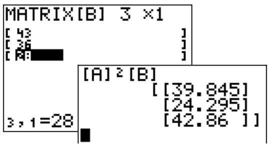
```
[B]*[A]
[[610 800 320 1…

[B]*[A]
… 800 320 1800]]

[A]*[C]
            [[385]
             [905]
             [850]]

[B]*[A]*[C]
          [[78500]]
```

D4a $M^2 = \text{naar}\begin{array}{c|ccc} & A & B & C \\ A & 0{,}555 & 0{,}37 & 0{,}095 \\ B & 0{,}245 & 0{,}32 & 0{,}8 \\ C & 0{,}2 & 0{,}31 & 0{,}825 \end{array}$. $m_{32} = 0{,}31$ van M^2 is de kans dat een leerling die nu agenda B gebruikt over twee jaar agenda C gebruikt.



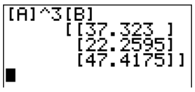
```
MATRIX[A] 3 ×3
[ .7   .3   .05  ]
[ .2   .5   .05  ]
[ .1   .2   .9   ]

[A]²
[[.555 .37 .095…
 [.245 .32 .08 …
 [.2   .31 .825…
```

D4b $M^2 \cdot \text{naar}\begin{array}{c|c} A & 43 \\ B & 36 \\ C & 28 \end{array} \approx \text{naar}\begin{array}{c|c} A & 40 \\ B & 24 \\ C & 43 \end{array} \Rightarrow$ in 2009-2010 gebruiken 40 leerlingen agenda A.



```
MATRIX[B] 3 ×1
[ 43  ]
[ 36  ]
[ 28  ]

3,1=28

[A]²[B]
      [[39.845]
       [24.295]
       [42.86 ]]
```

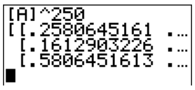
D4c $M^3 \cdot \text{naar}\begin{array}{c|c} A & 43 \\ B & 36 \\ C & 28 \end{array} \approx \text{naar}\begin{array}{c|c} A & 37 \\ B & 22 \\ C & 48 \end{array} \Rightarrow$ in 2010-2011 gebruiken 22 leerlingen agenda B.
(het aantal leerlingen blijft gelijk $\Rightarrow$ 47,4 afronden naar 48)



```
[A]^3[B]
      [[37.323 ]
       [22.2595]
       [47.4175]]
```

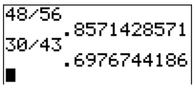
D4d

Voldoende grote n geeft: $M^n \approx \text{naar}\begin{array}{c|ccc} & A & B & C \\ A & 0{,}26 & 0{,}26 & 0{,}26 \\ B & 0{,}16 & 0{,}16 & 0{,}16 \\ C & 0{,}58 & 0{,}58 & 0{,}58 \end{array} \Rightarrow$ op den duur maakt 58% van de leerlingen gebruik van agenda C.

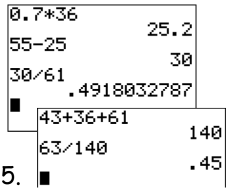


```
[A]^250
[[.2580645161 …
 [.1612903226 …
 [.5806451613 …
```

D5a $P(\text{I} \to \text{II}) = \frac{48}{56} \approx 0{,}86$; $P(\text{II} \to \text{III}) = \frac{30}{43} \approx 0{,}70$.
Gegeven $P(\text{III} \to \text{IV}) = 0{,}70$.
Dus van de 36 dieren uit klasse III zitten er twee jaar later $0{,}7 \cdot 36 = 25$ in klasse IV.
De rest $55 - 26 = 30$ komt uit klasse IV.
$P(\text{IV} \to \text{IV}) = \frac{30}{61} \approx 0{,}49$.
$43 + 36 + 61 = 140$ dieren in II, III en IV zorgen voor 63 nakomelingen $\Rightarrow$
$P(\text{II} \to \text{I}) = P(\text{II} \to \text{I}) = P(\text{II} \to \text{I}) = \frac{63}{140} = 0{,}45$.
Zie de populatievoorspellingsmatix L hiernaast.

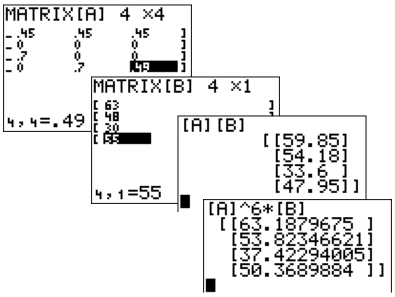


```
48/56
        .8571428571
30/43
        .6976744186
```



```
0.7*36
                25.2
55-25
                  30
30/61
        .4918032787

43+36+61
                 140
63/140
                 .45
```

$L = \text{naar}\begin{array}{c|cccc} & \text{I} & \text{II} & \text{III} & \text{IV} \\ \text{I} & 0 & 0{,}45 & 0{,}45 & 0{,}45 \\ \text{II} & 0{,}86 & 0 & 0 & 0 \\ \text{III} & 0 & 0{,}70 & 0 & 0 \\ \text{IV} & 0 & 0 & 0{,}70 & 0{,}49 \end{array}$ (van)

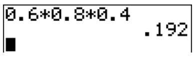


```
MATRIX[A] 4 ×4
[ .45  .45  .45  ]
[ 0    0    0    ]
[ .7   0    0    ]
[ 0    .7   .49  ]

4,4=.49

MATRIX[B] 4 ×1
[ 63  ]
[ 48  ]
[ 30  ]
[ 55  ]

4,1=55

[A][B]
      [[59.85]
       [54.18]
       [33.6 ]
       [47.95]]

[A]^6*[B]
 [[63.1879675 ]
  [53.82346621]
  [37.42294005]
  [50.3689884 ]]
```

nov. 2019

D5b $L \cdot \text{naar}\begin{array}{c|c} \text{I} & 63 \\ \text{II} & 48 \\ \text{III} & 30 \\ \text{IV} & 55 \end{array} \approx \text{naar}\begin{array}{c|c} \text{I} & 60 \\ \text{II} & 54 \\ \text{III} & 34 \\ \text{IV} & 48 \end{array}$ nov. 2009 en $L^6 \cdot \text{naar}\begin{array}{c|c} \text{I} & 63 \\ \text{II} & 48 \\ \text{III} & 30 \\ \text{IV} & 55 \end{array} \approx \text{naar}\begin{array}{c|c} \text{I} & 63 \\ \text{II} & 54 \\ \text{III} & 37 \\ \text{IV} & 50 \end{array}$.

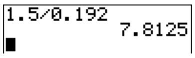
D6a Op grond van theorie C op bladzijde 173 krijg je: $(0{,}6 \cdot 0{,}8 \cdot 0{,}4 \cdot a = 0{,}192a)$



```
0.6*0.8*0.4
                .192
```

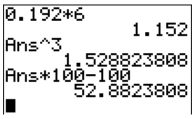
$L^4 = \text{naar}\begin{array}{c|cccc} & \text{I} & \text{II} & \text{III} & \text{IV} \\ \text{I} & 0{,}192a & 0 & 0 & 0 \\ \text{II} & 0 & 0{,}192a & 0 & 0 \\ \text{III} & 0 & 0 & 0{,}192a & 0 \\ \text{IV} & 0 & 0 & 0 & 0{,}192a \end{array}$. (van)

D6b De groeifactor is 1,5 (10% + 50% = 150%) $\Rightarrow 0{,}192a = 1{,}5 \Rightarrow a = \frac{1{,}5}{0{,}192} = 7{,}8125$.



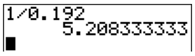
```
1.5/0.192
              7.8125
```

D6c $a = 6 \Rightarrow$ groeifactor per 4 jaar is $0{,}192 \cdot 6 = 1{,}152 \Rightarrow$ groeifactor per 12 jaar is $1{,}152^3 \approx 1{,}529$.
De populatie neemt per 12 jaar met (ongeveer) 52,9% toe.



```
0.192*6
               1.152
Ans^3
         1.528823808
Ans*100-100
         52.8823808
```

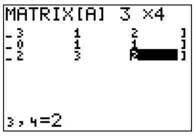
D6d De groeifactor is kleiner dan $1 \Rightarrow 0{,}192a < 1 \Rightarrow a < 5{,}2$.



```
1/0.192
         5.208333333
```

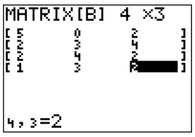
Gemengde opgaven 12. Grafen en matrices

G31a 💻 $$A = \begin{array}{c|cccc} & M_1 & M_2 & M_3 & M_4 \\ \hline G_1 & 2 & 3 & 1 & 2 \\ G_2 & 2 & 0 & 1 & 1 \\ G_3 & 1 & 2 & 3 & 2 \end{array}.$$



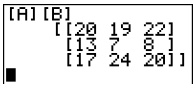
```
MATRIX[A] 3 ×4
[3      1      2      ]
[0      1      1      ]
[2      3      2      ]

3,4=2
```

G31b 💻 $$S = \begin{array}{c|ccc} & P_1 & P_2 & P_3 \\ \hline M_1 & 5 & 0 & 2 \\ M_2 & 2 & 3 & 4 \\ M_3 & 2 & 4 & 2 \\ M_4 & 1 & 3 & 2 \end{array}.$$



```
MATRIX[B] 4 ×3
[5      0      2      ]
[2      3      4      ]
[2      4      2      ]
[1      3      2      ]
4,3=2
```

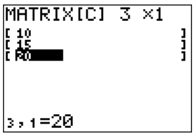
G31c 💻 $$C = A \cdot S = \begin{array}{c|ccc} & P_1 & P_2 & P_3 \\ \hline G_1 & 20 & 19 & 22 \\ G_2 & 13 & 7 & 8 \\ G_3 & 17 & 24 & 20 \end{array}.$$



```
[A][B]
    [[20 19 22]
     [13 7  8 ]
     [17 24 20]]
```

$c_{12} = 19 \Rightarrow$ voor één eenheid P_2 zijn 19 eenheden G_1 nodig.
$c_{31} = 17 \Rightarrow$ voor één eenheid P_1 zijn 17 eenheden G_3 nodig.

G31d 💻 $$B = \begin{array}{c|c} & \text{aant.} \\ \hline P_1 & 10 \\ P_2 & 15 \\ P_3 & 20 \end{array}.$$

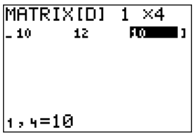


```
MATRIX[C] 3 ×1
[10                   ]
[15                   ]
[20                   ]

3,1=20
```

$S \cdot B$ heeft alleen zin als B uit 3 rijen bestaat.

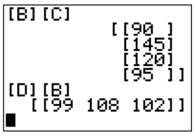
$$K = \begin{array}{c} M_1 \; M_2 \; M_3 \; M_4 \\ €(9 \;\; 10 \;\; 12 \;\; 10) \end{array}.$$

$K \cdot S$ heeft alleen zin als K uit 4 kolommen bestaat.



```
MATRIX[D] 1 ×4
[10     12     10     ]

1,4=10
```

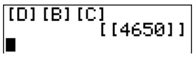
G31e 💻 $$S \cdot B = \begin{array}{c|c} & \text{aant.} \\ \hline M_1 & 90 \\ M_2 & 145 \\ M_3 & 120 \\ M_4 & 95 \end{array} \text{ en } K \cdot S = \begin{array}{c} P_1 \;\; P_2 \;\; P_3 \\ €(99 \;\; 108 \;\; 102) \end{array}.$$



```
[B][C]
          [[90 ]
           [145]
           [120]
           [95 ]]
[D][B]
   [[99 108 102]]
```

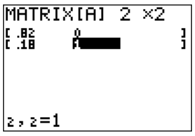
De elementen van $S \cdot B$ geven aan hoeveel eenheden van de verschillende mengsels er nodig zijn voor de bestelling.
De elementen van $K \cdot S$ geven de kosten per eenheid van de producten P_1, P_2 en P_3.

G31f 💻 $K \cdot S \cdot B = \overset{\text{aant.}}{€(4\,650)} \Rightarrow$ de kosten van de bestelling.



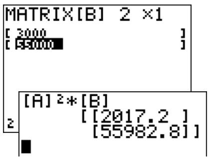
```
[D][B][C]
           [[4650]]
```

G32a 💻 $$V = \text{naar} \begin{array}{c|cc} & \multicolumn{2}{c}{\text{van}} \\ & G & A \\ \hline G & 0{,}82 & 0 \\ A & 0{,}18 & 1 \end{array}.$$



```
MATRIX[A] 2 ×2
[.82    0      ]
[.18    1      ]

2,2=1
```

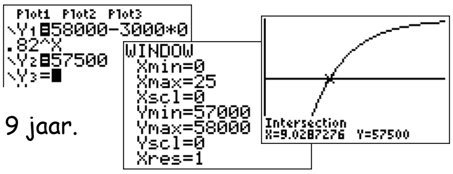
G32b 💻 $$V = \text{naar} \begin{array}{c|c} & \text{aantal} \\ \hline G & 3\,000 \\ R & 55\,000 \end{array} \text{ en } V^2 \cdot B = \text{naar} \begin{array}{c|c} & \text{aantal} \\ \hline G & 2\,017 \\ R & 55\,983 \end{array}.$$



```
MATRIX[B] 2 ×1
[3000                 ]
[55000                ]
2
[A]²*[B]
   [[2017.2 ]
    [55982.8]]
```

Na twee jaar wonen er 55983 mensen in rest van de stad.

G33a 💻 $N_g = 3\,000 \cdot 0{,}82^n$
$N_r = 58\,000 - 3\,000 \cdot 0{,}82^n$ (er wonen totaal 58 000 mensen in de stad).



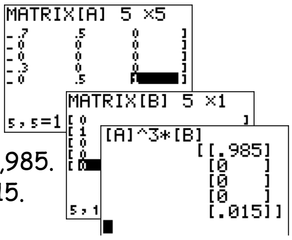
```
Plot1 Plot2 Plot3
\Y1■58000-3000*0
.82^X
\Y2■57500
\Y3=■
WINDOW
 Xmin=0
 Xmax=25
 Xscl=0
 Ymin=57000
 Ymax=58000
 Yscl=0
 Xres=1
Intersection
X=9.0287276  Y=57500
```

G33b 💻 $N_r = 58\,000 - 3\,000 \cdot 0{,}82^n = 57\,500$ (intersect) $\Rightarrow n \approx 9{,}03$. Dus na 9 jaar.

G34a 💻 $r_{13} = 0{,}7$ is de kans dat een klant van categorie C één maand later naar categorie A gaat.
(dus de kans dat een klant, wiens rekening tussen 1 en 2 maanden openstaat, alsnog binnen één maand betaalt)
$r_{32} = 0{,}1$ is de kans dat een klant van categorie B één maand later naar categorie C gaat.
(dus de kans dat een klant, wiens rekening minder dan 1 maand openstaat, binnen één maand weer niet betaalt)
$r_{52}=0$ is de kans dat een klant van categorie B één maand later naar categorie D gaat (dus C overslaat).
(dus de kans dat een klant, wiens rekening minder dan 1 maand openstaat, na één maand al tussen 2 en 3 maanden openstaat)

G34b 💻 Het is niet mogelijk om van een willekeurige categorie naar categorie B over te gaan.

G34c 💻 Een wanbetaler blijft wanbetaler (en wordt dus afgeboekt).

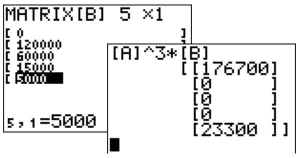


```
MATRIX[A] 5 ×5
[.7     .5     0      ]
[0      0      0      ]
[0      0      0      ]
[.3     0      0      ]
[0      .5     0      ]

5,5=1
MATRIX[B] 5 ×1
[0                    ]
[1
[0
[0
[0

5,1
[A]^3*[B]
         [[.985]
          [0   ]
          [0   ]
          [0   ]
          [.015]]
```

G34d 💻 $$R^3 \cdot K = \begin{array}{c|c} A & 0{,}985 \\ B & 0 \\ C & 0 \\ D & 0 \\ E & 0{,}015 \end{array}.$$ De kans dat een klant uit categorie B binnen 3 maanden betaalt, is 0,985.
De kans dat hij na 3 maanden als wanbetaler wordt afgeboekt is 0,015.
Andere mogelijkheden zijn er niet.

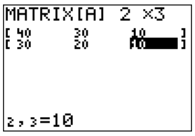
G34e 💻 $$R^3 \cdot B = R^3 \cdot \begin{array}{c|c} A & 0 \\ B & 120000 \\ C & 60000 \\ D & 15000 \\ E & 5000 \end{array} = \begin{array}{c|c} A & 176700 \\ B & 0 \\ C & 0 \\ D & 0 \\ E & 23\,300 \end{array}.$$

Uiteindelijk wordt € 23 300 afgeboekt.
Hiervoor wordt de deurwaarder ingeschakeld.

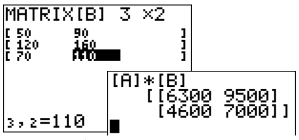


```
MATRIX[B] 5 ×1
[0                    ]
[120000
[60000
[15000
[5000

5,1=5000
[A]^3*[B]
       [[176700]
        [0     ]
        [0     ]
        [0     ]
        [23300 ]]
```

G35a 💻 $$V \cdot P = \begin{array}{c|ccc} & h & m & k \\ \hline \text{I} & 40 & 30 & 10 \\ \text{II} & 30 & 20 & 10 \end{array} \cdot \begin{array}{c|cc} & o & b \\ \hline h & 50 & 90 \\ m & 120 & 160 \\ k & 70 & 110 \end{array} = \begin{array}{c|cc} & o & b \\ \hline \text{I} & 6\,300 & 9\,500 \\ \text{II} & 4\,600 & 7\,000 \end{array}.$$



```
MATRIX[A] 2 ×3
[40     30     10     ]
[30     20     10     ]

2,3=10
```

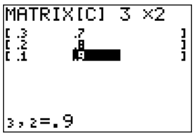


```
MATRIX[B] 3 ×2
[50     90     ]
[120    160    ]
[70     110    ]

3,2=110
[A]*[B]
   [[6300 9500]
    [4600 7000]]
```

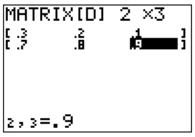
De getallen geven per filiaal de waarde in euro's van de voorraad met en zonder bespanning.

G35b 💻 Als alle onbespannen rackets verkocht worden, levert dat $6\,300 + 4\,600 = 10\,900$ euro op.

G35c 💻 $G = \begin{array}{c|cc|l} & o & b & \\ h & 0{,}3 & 0{,}7 & \rightarrow \text{samen } 1 \\ m & 0{,}2 & 0{,}8 & \rightarrow \text{samen } 1 \\ k & 0{,}1 & 0{,}9 & \rightarrow \text{samen } 1 \end{array}$ en $H = \begin{array}{c|ccc|} & h & m & k \\ o & 0{,}3 & 0{,}2 & 0{,}1 \\ b & 0{,}7 & 0{,}8 & 0{,}9 \end{array}$.

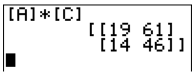


```
MATRIX[C] 3 ×2
[ .3      .7      ]
[ .2      .8      ]
[ .1      .9      ]

3,2=.9
```



```
MATRIX[D] 2 ×3
[ .3      .2      .1 ]
[ .7      .8      .9 ]

2,3=.9
```

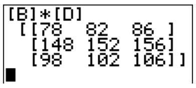
G35d 💻 $V \cdot G = \begin{array}{c|cc|} & o & b \\ \text{I} & 19 & 61 \\ \text{II} & 14 & 46 \end{array}$.



```
[A]*[C]
         [[19 61]
          [14 46]]
▮
```

De getallen geven per filiaal de aantallen rackets die onbespannen en bespannen verkocht worden.

G35e 💻 $P \cdot H = \begin{array}{c|ccc|} & h & m & k \\ h & 78 & - & - \\ m & - & 152 & - \\ k & - & - & 106 \end{array}$.



```
[B]*[D]
 [[78  82  86 ]
  [148 152 156]
  [98  102 106]]
▮
```

De getallen op de hoofddiagonaal geven de gemiddelde opbrengst van elk soort racket. (de andere getallen zijn onzin)

G36a 💻 Het element $l_{99} \neq 0$.

G36b 💻 $l_{21} = \frac{1068}{1090} \approx 0{,}98$

$l_{32} = \frac{804}{820} \approx 0{,}98$

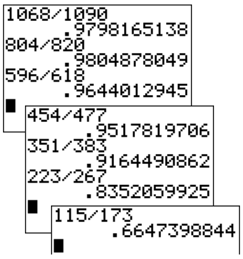
$l_{43} = \frac{596}{618} \approx 0{,}96$

$l_{54} = \frac{454}{477} \approx 0{,}95$

$l_{65} = \frac{351}{383} \approx 0{,}92$

$l_{76} = \frac{223}{267} \approx 0{,}84$

$l_{87} = \frac{115}{173} \approx 0{,}66$.



```
1068/1090
       .9798165138
804/820
       .9804878049
596/618
       .9644012945
▮
```

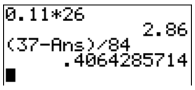
```
454/477
       .9517819706
351/383
       .9164490862
223/267
       .8352059925
▮
```

```
115/173
       .6647398844
▮
```

$$L = \begin{array}{c|ccccccccc|} & \text{0-9} & \text{9-19} & \text{20-29} & \text{30-39} & \text{40-49} & \text{50-59} & \text{60-69} & \text{70-79} & \text{80+} \\ \text{0-9} & 0 & 0{,}52 & 0{,}83 & 0{,}45 & 0{,}08 & 0 & 0 & 0 & 0 \\ \text{10-19} & 0{,}98 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \text{20-29} & 0 & 0{,}98 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \text{30-39} & 0 & 0 & 0{,}96 & 0 & 0 & 0 & 0 & 0 & 0 \\ \text{40-49} & 0 & 0 & 0 & 0{,}95 & 0 & 0 & 0 & 0 & 0 \\ \text{50-59} & 0 & 0 & 0 & 0 & 0{,}92 & 0 & 0 & 0 & 0 \\ \text{60-69} & 0 & 0 & 0 & 0 & 0 & 0{,}84 & 0 & 0 & 0 \\ \text{70-79} & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}66 & 0 & 0 \\ \text{80+} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0{,}41 & 0{,}11 \end{array}.$$

Van de 26 (duizend) 80+vrouwen in 1960 leven er nog $0{,}11 \cdot 26 \approx 3$ (duizend) in 1970.

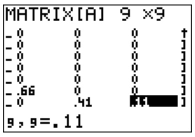
Dus $l_{98} = \frac{37 - \text{Ans}}{84} \approx 0{,}41$. Samen met de gegevens over D levert dit de Lesliematrix L hierboven.



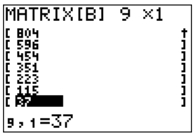
```
0.11*26
              2.86
(37-Ans)/84
       .4064285714
▮
```

G36c 💻

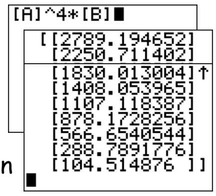
$$L^4 \cdot \begin{array}{c|c|} & \text{aantal} \\ \text{0-9} & 1264 \\ \text{10-19} & 1068 \\ \text{20-29} & 804 \\ \text{30-39} & 596 \\ \text{40-49} & 454 \\ \text{50-59} & 351 \\ \text{60-69} & 223 \\ \text{70-79} & 115 \\ \text{80+} & 37 \end{array} = \begin{array}{c|c|} & \text{aantal} \\ \text{0-9} & 2789 \\ \text{10-19} & 2251 \\ \text{20-29} & 1830 \\ \text{30-39} & 1408 \\ \text{40-49} & 1107 \\ \text{50-59} & 878 \\ \text{60-69} & 567 \\ \text{70-79} & 289 \\ \text{80+} & 105 \end{array}.$$



```
MATRIX[A] 9 ×9
-0       0       0    †
-0       0       0    ]
-0       0       0    ]
-0       0       0    ]
-0       0       0    ]
-.66     0       0    ]
-0       .41     .11  ]
9,9=.11
```



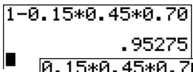
```
MATRIX[B] 9 ×1
[ 804             †
[ 596             ]
[ 454             ]
[ 351             ]
[ 223             ]
[ 115             ]
[ 37              ]
9,1=37
```



```
[A]^4*[B]▮
 [[2789.194652]
  [2250.711402]
  [1830.013004]↑
  [1408.053965]
  [1107.118387]
  [878.1728256]
  [566.6540544]
  [288.7891776]
  [104.514876 ]]
▮
```

De laatste kolom (×1000) geeft een voorspelling van de samenstelling van de vrouwelijke bevolking van Chili in 2010.

G37a 💻 $P(\text{minstens één keer uitgeleend}) = 1 - P(\text{geen keer uitgeleend}) = 1 - 0{,}15 \cdot 0{,}45 \cdot 0{,}70 = 0{,}95275 \approx 0{,}95$.

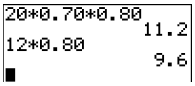


```
1-0.15*0.45*0.70
            .95275
▮
```

G37b 💻 $P(\text{nog nooit uitgeleend}) = 0{,}15 \cdot 0{,}45 \cdot 0{,}70 \cdot 0{,}80 = 0{,}0378 \approx 0{,}04$.

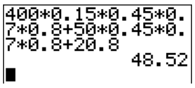
```
0.15*0.45*0.70*0
.80
             .0378
▮
```

G37c 💻 Van de 20 boeken zijn het er $20 \cdot 0{,}70 \cdot 0{,}80$ en van de 12 boeken $12 \cdot 0{,}8$.
Totaal zijn dat $11{,}2 + 9{,}6 = 20{,}8 \approx 21$ boeken.



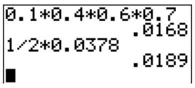
```
20*0.70*0.80
              11.2
12*0.80
               9.6
▮
```

G37d 💻 $400 \cdot 0{,}15 \cdot 0{,}45 \cdot 0{,}70 \cdot 0{,}80 + 50 \cdot 0{,}45 \cdot 0{,}70 \cdot 0{,}80 + 20 \cdot 0{,}70 \cdot 0{,}80 + 12 \cdot 0{,}8 = 48{,}52 \approx 49$ boeken.



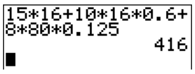
```
400*0.15*0.45*0.
7*0.8+50*0.45*0.
7*0.8+20.8
             48.52
▮
```

G37e 💻 Het deel dat eerst onuitgeleend uit de roulatie werd genomen is 0,0378 (zie vraag b).
In de nieuwe situatie wordt dat $0{,}1 \cdot 0{,}4 \cdot 0{,}6 \cdot 0{,}7 = 0{,}0168$.
Omdat $\frac{1}{2} \cdot 0{,}0378 = 0{,}0189$ en $0{,}0168 < 0{,}0189$ heeft de directeur gelijk.

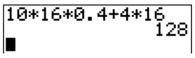


```
0.1*0.4*0.6*0.7
             .0168
1/2*0.0378
             .0189
▮
```

G38a 💻 Vanille-ijs: $15 \cdot 16 + 10 \cdot 16 \cdot 0{,}16 + 8 \cdot 80 \cdot 0{,}125 = 416$ liter.
Aardbeienijs: $10 \cdot 16 \cdot 0{,}4 + 4 \cdot 16 = 128$ liter.

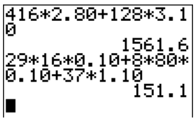


```
15*16+10*16*0.6+
8*80*0.125
               416
▮
```

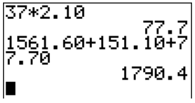


```
10*16*0.4+4*16
               128
▮
```

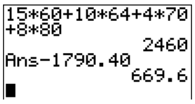
G38b 💻 Grondstofkosten: $416 \cdot 2{,}80 + 128 \cdot 3{,}10 = 1561{,}60$ euro.
Verpakkingskosten: $(15 + 10 + 4) \cdot 16 \cdot 0{,}10 + 8 \cdot 80 \cdot 0{,}10 + (15 + 10 + 4 + 8) \cdot 1{,}10 = 151{,}10$ euro.
Transportkosten: $(15 + 10 + 4 + 8) \cdot 2{,}10 = 77{,}70$ euro.
Totale kosten: $1561{,}60 + 151{,}10 + 77{,}70 = 1790{,}40$ euro.
Inkomsten: $15 \cdot 60 + 10 \cdot 64 + 4 \cdot 70 + 8 \cdot 80 = 2\,460$ euro.
Winst: $2\,460 - 1790{,}40 = 669{,}60$ euro.



```
416*2.80+128*3.1
0
            1561.6
29*16*0.10+8*80*
0.10+37*1.10
             151.1
▮
```



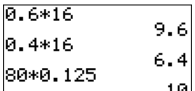
```
37*2.10
              77.7
1561.60+151.10+7
7.70
            1790.4
▮
```



```
15*60+10*64+4*70
+8*80
              2460
Ans-1790.40
             669.6
▮
```

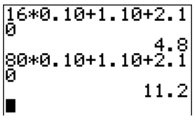
G38c 💻 M is een 4×2-matrix met op de bovenste rij de liters vanille-ijs en op de onderste rij de liters aardbeienijs per doos.

Dus $M = \begin{array}{c|cccc|} & v & d & a & p \\ V & 16 & 9{,}6 & 0 & 10 \\ A & 0 & 6{,}4 & 16 & 0 \end{array}$.



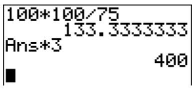
```
0.6*16
               9.6
0.4*16
               6.4
80*0.125
                10
▮
```

G38d 🖥 $W = (A - B \cdot M - C) \cdot P = A \cdot P - B \cdot M \cdot P - C \cdot P$.
$A \cdot P$ geeft de inkomsten en $C \cdot P$ geeft de verpakkingskosten en transportkosten.
Dus in $B \cdot M \cdot P$ staan de grondstofkosten. Voor B krijg je matrix hiernaast. $B = \begin{pmatrix} \overset{v}{2{,}80} & \overset{a}{3{,}10} \end{pmatrix}$.
$M \cdot P = 2 \times 4\text{-matrix} \cdot 4 \times 1\text{-matrix} = 2 \times 1\text{-matrix}$ en $B \cdot (M \cdot P) = ? \times 2\text{-matrix} \cdot 2 \times 1\text{-matrix} = 1 \times 1\text{-matrix} \Rightarrow ? = 1$.



```
16*0.10+1.10+2.1
0
                4.8
80*0.10+1.10+2.1
0
               11.2
```

G39a 🖥 Per schildpad komen er 100 eieren uit.
Dus $75\% = 100 \Rightarrow 100\% = 100 \cdot 1\% = 100 \cdot \frac{100}{75} \approx 133$ eieren gemiddeld per schildpad.
Per vrouwtje $3 \cdot \text{Ans} = 400$ eieren.

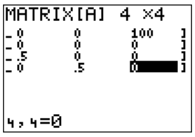


```
100*100/75
        133.3333333
Ans*3
                400
```

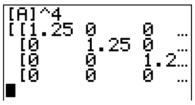
G39b 🖥

van

$$L^4 = \text{naar} \begin{array}{c|cccc} & \text{I} & \text{II} & \text{III} & \text{IV} \\ \text{I} & 1{,}25 & 0 & 0 & 0 \\ \text{II} & 0 & 1{,}25 & 0 & 0 \\ \text{III} & 0 & 0 & 1{,}25 & 0 \\ \text{IV} & 0 & 0 & 0 & 1{,}25 \end{array}$$



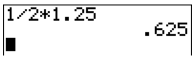
```
MATRIX[A]  4 ×4
[ 0      0      100  ]
[ 0      0      0    ]
[ .5     0      0    ]
[ 0      .5     0    ]

4,4=0
```



```
[A]^4
[[1.25 0    0   …
 [0    1.25 0   …
 [0    0    1.2…
 [0    0    0   …
```

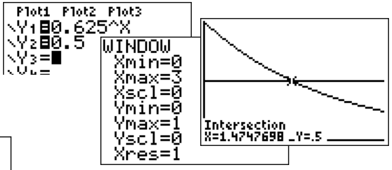
Per 100 jaar (4 perioden van 25 jaar) een toename van 25%.

G39c 🖥 Op de hoofdiagonaal van L^4 komt $\frac{1}{2} \cdot 1{,}25 = 0{,}625$.



```
1/2*1.25
              .625
```

$0{,}625^t = 0{,}5$ (intersect) $\Rightarrow t \approx 1{,}47$.
Na $1{,}47 \cdot 100 = 147$ jaar is nog de helft van het oorspronkelijke aantal over.



```
 Plot1  Plot2  Plot3
\Y1=0.625^X
\Y2=0.5
\Y3=
```

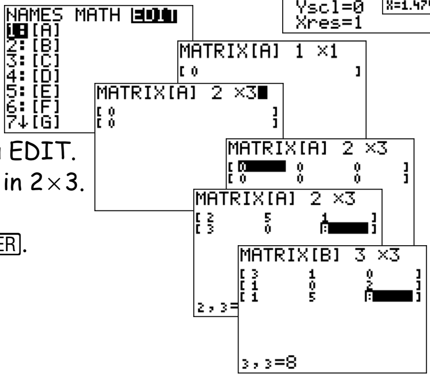
```
WINDOW
 Xmin=0
 Xmax=3
 Xscl=0
 Ymin=0
 Ymax=1
 Yscl=0
 Xres=1
```

```
Intersection
X=1.4747698  Y=.5
```

TI-84 13. Matrices

De matrix $A = \begin{pmatrix} 2 & 5 & 1 \\ 3 & 0 & 8 \end{pmatrix}$ voer je als volgt op de GR in.

- Kies $\boxed{\text{MATRIX}}$ ($= \boxed{\text{2nd}}\boxed{x^{-1}}$) en ga met $\boxed{\triangleright}$ of $\boxed{\triangleleft}$ naar het menu EDIT.
- Kies 1: [A] en verander op de bovenste regel de afmeting in 2×3.
  Na krijg je het vierde scherm hiernaast.
- Tik in $\boxed{2}\boxed{\text{ENTER}}$ $\boxed{5}\boxed{\text{ENTER}}$ $\boxed{1}\boxed{\text{ENTER}}$ $\boxed{3}\boxed{\text{ENTER}}$ $\boxed{0}\boxed{\text{ENTER}}$ en $\boxed{8}\boxed{\text{ENTER}}$.
  Je hebt matrix A ingevoerd.



```
NAMES MATH EDIT
1:[A]
2:[B]
3:[C]
4:[D]
5:[E]
6:[F]
7↓[G]
```

```
MATRIX[A]  1 ×1
[ 0              ]
```

```
MATRIX[A]  2 ×3
[ 0              ]
[ 0              ]
```

```
MATRIX[A]  2 ×3
[ 0      0      0    ]
[ 0      0      0    ]
```

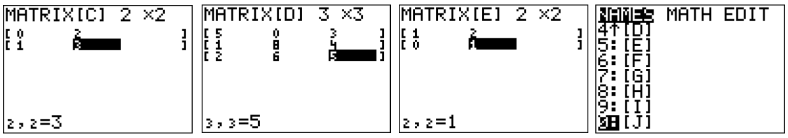
```
MATRIX[A]  2 ×3
[ 2      5      1    ]
[ 3      0      8    ]

2,3=
```

```
MATRIX[B]  3 ×3
[ 3      1      0    ]
[ 1      0      2    ]
[ 1      5      8    ]

3,3=8
```

Voer nu ook de matrix $B = \begin{pmatrix} 3 & 1 & 0 \\ 1 & 0 & 2 \\ 1 & 5 & 8 \end{pmatrix}$ op de GR in.

Heb je een fout gemaakt bij het invoeren, dan ga je met de cursor naar de fout, waarna de fout te herstellen is.

🖩1a Zie de schermen hiernaast.
🖩1b 10.
🖩1c *
🖩1d *



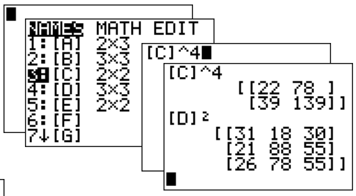
```
MATRIX[C]  2 ×2
[ 0      2       ]
[ 1      3       ]

2,2=3
```

```
MATRIX[D]  3 ×3
[ 5      0      3    ]
[ 1      8      4    ]
[ 2      6      5    ]

3,3=5
```

```
MATRIX[E]  2 ×2
[ 1      2       ]
[ 0      1       ]

2,2=1
```

```
NAMES MATH EDIT
4↑[D]
5:[E]
6:[F]
7:[G]
8:[H]
9:[I]
0:[J]
```

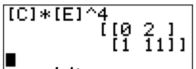
🖩2a $C^4 = \begin{pmatrix} 22 & 78 \\ 39 & 139 \end{pmatrix}$ en $D^2 = \begin{pmatrix} 31 & 18 & 30 \\ 21 & 88 & 55 \\ 26 & 78 & 55 \end{pmatrix}$.



```
NAMES MATH EDIT
1:[A]  2×3
2:[B]  3×3
3:[C]  2×2
4:[D]  3×3
5:[E]  2×2
6:[F]
7↓[G]
```

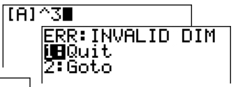
```
[C]^4
        [[22  78 ]
         [39  139]]
[D]²
      [[31  18  30]
       [21  88  55]
       [26  78  55]]
```

🖩2b $C \cdot E^4 = \begin{pmatrix} 0 & 2 \\ 1 & 11 \end{pmatrix}$.



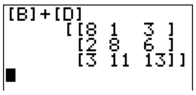
```
[C]*[E]^4
          [[0  2 ]
           [1  11]]
```

🖩2c A^3 geeft een foutmelding, want A is geen vierkante matrix.



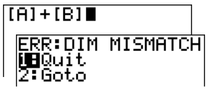
```
[A]^3
ERR:INVALID DIM
1:Quit
2:Goto
```

🖩3a $B + D = \begin{pmatrix} 8 & 1 & 3 \\ 2 & 8 & 6 \\ 3 & 11 & 13 \end{pmatrix}$.



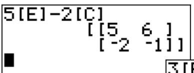
```
[B]+[D]
        [[8  1   3 ]
         [2  8   6 ]
         [3  11  13]]
```

🖩3b $A + B$ geeft een foutmelding, want A en B hebben niet dezelfde afmeting.



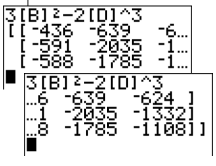
```
[A]+[B]
ERR:DIM MISMATCH
1:Quit
2:Goto
```

🖩3c $5E - 2C = \begin{pmatrix} 5 & 6 \\ -2 & -1 \end{pmatrix}$.



```
5[E]-2[C]
          [[5   6 ]
           [-2  -1]]
```

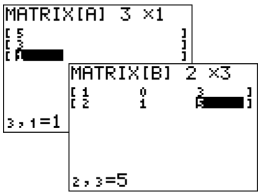
🖩3d $3B^2 - 2D^3 = \begin{pmatrix} -436 & -639 & -624 \\ -591 & -2035 & -1332 \\ -588 & -1785 & -1108 \end{pmatrix}$.



```
3[B]²-2[D]^3
[[-436  -639   -6…
 [-591  -2035  -1…
 [-588  -1785  -1…
```

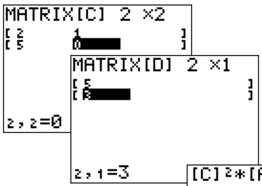
```
3[B]²-2[D]^3
…6  -639   -624 ]
…1  -2035  -1332]
…8  -1785  -1108]]
```

🖩4a Zie de schermen hiernaast.



```
MATRIX[A]  3 ×1
[ 5              ]
[ 3              ]
[ 1              ]

3,1=1
```

```
MATRIX[B]  2 ×3
[ 1      0      3    ]
[ 2      1      6    ]

2,3=5
```

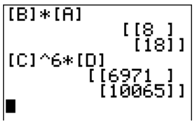


```
MATRIX[C]  2 ×2
[ 2      1       ]
[ 5      0       ]

2,2=0
```

```
MATRIX[D]  2 ×1
[ 5              ]
[ 3              ]

2,1=3
```

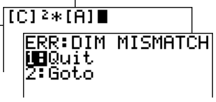
🖩4b $B \cdot A = \begin{pmatrix} 8 \\ 18 \end{pmatrix}$.

🖩4c $C^6 \cdot D = \begin{pmatrix} 6971 \\ 10065 \end{pmatrix}$.



```
[B]*[A]
              [[8 ]
               [18]]
[C]^6*[D]
          [[6971 ]
           [10065]]
```

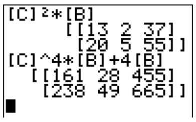
🖩4d $C^2 \cdot A$ geeft een foutmelding want A en B hebben niet dezelfde afmeting.



```
[C]²*[A]
ERR:DIM MISMATCH
1:Quit
2:Goto
```

$C^2 \cdot B = \begin{pmatrix} 13 & 2 & 37 \\ 20 & 5 & 55 \end{pmatrix}$.

$C^4 \cdot B + 4B = \begin{pmatrix} 161 & 28 & 455 \\ 238 & 49 & 655 \end{pmatrix}$.



```
[C]²*[B]
      [[13  2  37]
       [20  5  55]]
[C]^4*[B]+4[B]
  [[161  28  455]
   [238  49  665]]
```